MASONRY FAILURE CRITERION UNDER BIAXIAL STRESS STATE

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ABSTRACT: Masonry is a material that exhibits distinct directional properties because the mortar joints act as planes of weakness. To define failure under biaxial stress, a 3D surface in terms of the two normal stresses and shear stress (or the two principal stresses and their orientation to the bed joints) is required. This paper describes a method to define a general anisotropic (orthotropic) failure surface of masonry under biaxial stress, using a cubic tensor polynomial. The evaluation of strength parameters is performed using existing experimental data through a least-squares approach. The validity of the method is demonstrated by comparing the derived failure surface with classical experimental results. The ability to ensure the closed shape of the failure surface, the unique mathematical form for all possible combinations of plane stress, and the satisfactory approximation with the results of the real masonry behavior under failure conditions are some of the advantages of the proposed method.

INTRODUCTION

Masonry, one of the older structural materials, has a mechanical behavior that has not yet been fully investigated. Only recently have there been systematic experimental and/or analytical investigations on the response of masonry and its failure modes.

Taking into account the numerous uncertainties of the problem, an analytical mathematical model describing the masonry failure surface in a simple manner should be an efficient tool for the investigation of the behavior of masonry structures. Many analytical criteria for masonry structures have already been proposed (Dhanasekar et al. 1985; Naraine and Sinha 1991; Scarpas 1991; Syrmakezis et al. 1995, 1997, 1999).

Experimental investigations can also be considered as an important support to the aforementioned efforts (Samarasinghe 1980; Page 1980, 1981; Tassios and Vachliotis 1989).

The aim of this paper is to introduce an anisotropic (orthotropic) failure surface under biaxial stress for masonry, using a cubic tensor polynomial. The agreement of this model with test data (Page 1981) clearly supports the use of the cubic tensor polynomial as a masonry failure surface under biaxial plane stress.

The method is presented in both a simple form and a general form. In both cases, the following characteristics of the polynomial have been proposed:

- It ensures the closed shape of the failure surface.
- It is to be expressed in a unique mathematical form for all possible combinations of plane stress. Note that the use of a failure surface that consists of more than one type of surface could demand additional effort in the analysis process of the masonry structure (Zienkiewicz and Taylor 1991). According to Zienkiewicz et al. (1969), the computation of singular points (''corners'') on failure surfaces may be avoided by a suitable choice of a continuous surface, which usually can represent, with a good degree of accuracy, the true condition.
- It leads to results in satisfactory approximation with the results of the existing real masonry behavior (experimental data) under failure conditions.
- It ensures the convex shape of the failure surface. According to Hill (1950) and Prager (1959), the failure surface for a stable material must be convex. This, in mathematical terms, is valid if the total Gaussian curvature K of the failure surface is positive.

PREVIOUS RESEARCH

Masonry exhibits distinct directional properties due to the influence of the mortar joints. Depending upon the orientation of the joints to the stress directions, failure can occur in the joints alone or simultaneously in the joints and blocks.

The failure of masonry under axiaxial and biaxial stress states has been studied experimentally in the past by many researchers, but there have been few attempts to obtain a general analytical failure criterion. The following is a brief review of the most representative experimental and analytical investigations.

Experimental Investigations

Researchers have long been aware of the significance of the bed joint angle to the applied load. Johnson and Thompson (1969) carried out compression tests on brick masonry disks to produce indirect tensile stresses on joints inclined at various angles to the vertical compressive load. Differences in failure patterns of the specimens were evident with the disk bed joints at various angles. The highest strength of the masonry has been observed for the cases when the compressive load was perpendicular to the bed joints or when the principal tensile stress at the center of the disk was parallel to the bed joints. In these cases failure occurred through bricks and perpendicular joints. The lowest strength has been observed when the compressive load was parallel to the bed joints or when the principal tensile stress at the center of the disk was perpendicular to the bed joints. In these cases failure occurred along the interface of the brick and mortar joint.

Similar tests have been reported by Page (1981). These experimental results, referring to a total of 102 panels, are used in this paper to demonstrate the validity of the method presented, and they will be extensively discussed. Ratios of vertical compressive stress σ_1 to horizontal compressive stress σ_2 of ∞ (uniaxial σ_1), 10, 4, 2, and 1 have been used in conjunction with a bed joint angle θ, with respect to the σ_1, in directions of 0°, 22.5°, 45°, 67.5°, and 90°. A minimum of four tests were performed for each combination of σ_1, σ_2, and θ. The failure envelopes that Page obtained by plotting mean curves for each bed joint angle are shown in Figs. 1–3. These curves will be used in this paper in comparisons with the results of the proposed analytical process.

A specific category of experimental investigations refers to the most critical part of the failure envelope of masonry, under

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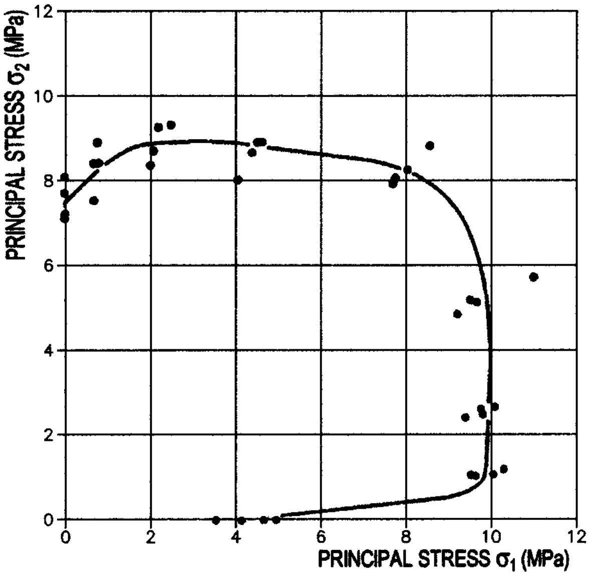

FIG. 1. Failure of Brickwork under Biaxial Compression (θ = 0°) (Page 1981)

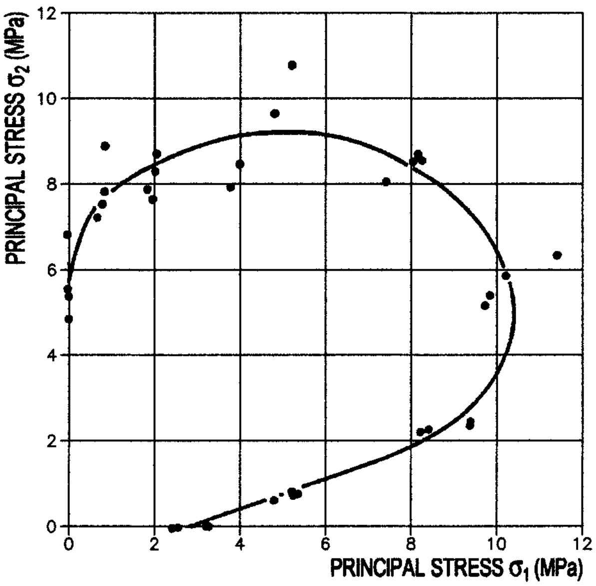

FIG. 2. Failure of Brickwork under Biaxial Compression (θ = 22.5°) (Page 1981)

heterosemous biaxial stresses. Such a biaxial stress state appears often in the case of in-plane loaded walls (e.g., under seismic loading or under differential settlements). The masonry failure surface for this case has been investigated by Tassios and Vachliotis (1989). To facilitate the experimental study of full-scale masonry under plane heterosemous stress conditions, a double diagonal compression test has been conceived. Using this tool, a number of tests have been carried out in the Laboratory of Reinforced Concrete of the National Technical University of Athens. The angle between the direction of compressive stress and the bed joint has been kept constant at 45°. This angle seems to present the highest practical interest for the verification under conditions of seismic and differential settlements. Similar to some of the curves cited in the literature (Samarasinghe and Hendry 1980; Page 1982), the shape of the obtained critical curve shows, in general, the unrealistic nature of a linear failure criterion for masonry and the heterosemous biaxial stress state.

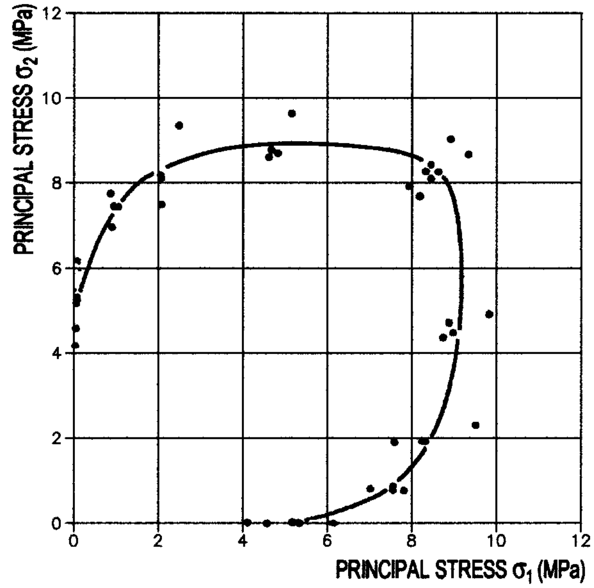

FIG. 3. Failure of Brickwork under Biaxial Compression (θ = 45°) (Page 1981)

Analytical Investigations

A failure surface for brick masonry in the tension-tension principal stresss region has been derived analytically by Page (1980). The shape of this failure surface was found to be critically dependent on the bed joint orientation and the relationship between the shear and tensile bond strengths of the mortar joints.

There have been few attempts to obtain a general failure criterion for masonry because of the difficulties in developing a representative biaxial test and the large number of tests involved. The problem has been discussed by Yokel and Fatal (1976) with reference to the failure of shear walls. Dhanasekar et al. (1985) interpolated the test data of Page (1981) by means of three elliptic cones; however, as the authors mentioned, the cones do not correspond with the observed distinct modes of failure. The elliptic cones have been expressed by a second-order tensor polynomial. This form of polynomial has also been used by Scarpas (1991) to define a general failure criterion for masonry. For both criteria, a number of uniaxial compression tests as well as biaxial tests are needed to determine the coefficients of the polynomial. A wide review of the subject can be found in Samarasinghe and Hendry (1980) and Hendry (1990).

ANALYTICAL MODEL

For the expression of an analytical failure model of masonry, a polynomial that is available already for composite materials is proposed. This failure surface in the stress space can be described by the equation (Tsai et al. 1971; Wu 1972; Jiang and Tennyson 1989)

$$f(\sigma_\ell) = F_i\sigma_i + F_{ij}\sigma_i\sigma_j + F_{ijk}\sigma_i\sigma_j\sigma_k + \cdots - 1 = 0 \qquad (1)$$

in which σ_ℓ = components of stresses (ℓ = 1, 2, . . . , 6); and F_i, F_{ij}, and F_{ijk} = coefficients to be properly determined (i, j, and k = 1, 2, . . . , 6).

If one restricts the analysis to a plane stress state and considers that a cubic formation is a reasonably accurate representation of the failure surface, then (1) reduces to

$$
\begin{aligned}
&F_1\sigma_1 + F_2\sigma_2 + F_6\sigma_6 + F_{11}\sigma_1^2 + F_{12}\sigma_1\sigma_2 + F_{16}\sigma_1\sigma_6 + F_{21}\sigma_2\sigma_1 \\
&\quad + F_{22}\sigma_2^2 + F_{26}\sigma_2\sigma_6 + F_{61}\sigma_6\sigma_1 + F_{62}\sigma_6\sigma_2 + F_{66}\sigma_6^2 + F_{111}\sigma_1^3 \\
&\quad + F_{112}\sigma_1^2\sigma_2 + F_{116}\sigma_1^2\sigma_6 + F_{121}\sigma_1^2\sigma_2 + F_{122}\sigma_1\sigma_2^2 + F_{126}\sigma_1\sigma_2\sigma_6 \\
&\quad + F_{161}\sigma_1^2\sigma_6 + F_{162}\sigma_1\sigma_2\sigma_6 + F_{166}\sigma_1\sigma_6^2 + F_{211}\sigma_2\sigma_1^2 + F_{212}\sigma_1\sigma_2^2 \\
&\quad + F_{216}\sigma_1\sigma_2\sigma_6 + F_{221}\sigma_1\sigma_2^2 + F_{222}\sigma_2^3 + F_{226}\sigma_2^2\sigma_6 + F_{261}\sigma_1\sigma_2\sigma_6 \\
&\quad + F_{262}\sigma_2^2\sigma_6 + F_{266}\sigma_2\sigma_6^2 + F_{611}\sigma_1^2\sigma_6 + F_{612}\sigma_1\sigma_2\sigma_6 + F_{616}\sigma_1\sigma_6^2 \\
&\quad + F_{621}\sigma_1\sigma_2\sigma_6 + F_{622}\sigma_2^2\sigma_6 + F_{626}\sigma_2\sigma_6^2 + F_{661}\sigma_1\sigma_6^2 + F_{662}\sigma_2\sigma_6^2 \\
&\quad + F_{666}\sigma_6^3 - 1 = 0 \qquad (2)
\end{aligned}
$$

The following assumptions have been made (Wu and Scheublein 1974):

- Symmetry of the material is assumed by the identity of "symmetric" coefficients, for $i \neq j \neq k \neq i$; that is, $F_{ijk} = F_{ikj} = F_{jik} = F_{kij} = F_{kji} = F_{jki}$, and $F_{ij} = F_{ji}$.
- The material under a given shear loading prossesses a common shear strength ($S = S'$) for both the positive direction and the negative direction of shear loading. Consequently, assuming that there is no dependence on the shear loading direction, the terms with odd exponents of σ_6 can be eliminated.
- The redundant terms F_{iii} (for i = 1, 2, and 6) are omitted (Wu and Scheublein 1974).

Using the notations (σ_x, σ_y, and τ) instead of (σ_1, σ_2, and σ_6), (1) takes the form

$$
\begin{aligned}
f(\sigma_x, \sigma_y, \tau) &= F_1\sigma_x + F_2\sigma_y + F_{11}\sigma_x^2 + F_{22}\sigma_y^2 + F_{66}\tau^2 + 2F_{12}\sigma_x\sigma_y \\
&\quad + 3F_{112}\sigma_x^2\sigma_y + 3F_{122}\sigma_x\sigma_y^2 + 3F_{166}\sigma_x\tau^2 + 3F_{266}\sigma_y\tau^2 - 1 = 0 \qquad (3)
\end{aligned}
$$

EVALUATION OF STRENGTH PARAMETERS

Principal Strength Tensor Components F_i, F_{ii}

Concerning factors F_i and F_{ii}, these can be determined using the experimental monoaxial tensile and compressive failure stresses across the x- and y-axes, respectively, as well as the shear failure stresses in the x, y-plane.

Monoaxial strengths of the wall in tension and compression across the x-axis are used and noted as X and X', respectively. For the case of masonry, the two points, (X, 0, 0) and ($-X'$, 0, 0), intersecting the x-axis and the failure surface are determined. For these points, (3) takes the form

$$F_1X + F_{11}X^2 = 1; \quad -F_1X' + F_{11}X'^2 = 1 \qquad (4)$$

The solution of the system of (4) gives the values

$$F_1 = \frac{1}{X} - \frac{1}{X'}; \quad F_{11} = \frac{1}{XX'} \qquad (5)$$

The monoxial tests across the y-axis leads, respectively, to the values of

$$F_2 = \frac{1}{Y} - \frac{1}{Y'}; \quad F_{22} = \frac{1}{YY'} \qquad (6)$$

The points of the failure surface, (0, 0, S) and (0, 0, $-S$), are determined by the test of the masonry panel in pure shear. Using (3), the result for these points is

$$F_{66} = \frac{1}{S^2} \qquad (7)$$

Interaction Strength Tensor Components F_{ij}, F_{ijk}

To determine the masonry failure criterion under the biaxial stress rate [(3)], values of constants F_{12}, F_{112}, F_{122}, F_{166}, and F_{266} have to be determined using the least-squares method. The constants are calculated through the system of equations

$$\frac{\partial E_\nu}{\partial F_{12}} = 0; \quad \frac{\partial E_\nu}{\partial F_{112}} = 0; \quad \frac{\partial E_\nu}{\partial F_{122}} = 0; \quad \frac{\partial E_\nu}{\partial F_{166}} = 0; \quad \frac{\partial E_\nu}{\partial F_{266}} = 0 \qquad (8)$$

where

$$
\begin{aligned}
E_\nu &= \sum_{i=1}^{\nu} (F_1\sigma_{xi} + F_2\sigma_{yi} + F_{11}\sigma_{xi}^2 + F_{22}\sigma_{yi}^2 + F_{66}\tau_i^2 + 2F_{12}\sigma_{xi}\sigma_{yi} \\
&\quad + 3F_{112}\sigma_{xi}^2\sigma_{yi} + 3F_{166}\sigma_{xi}\tau_i^2 + 3F_{266}\sigma_{yi}\tau_i^2 - 1)^2 \qquad (9)
\end{aligned}
$$

The equations to be used for ν groups of values (σ_{xi}, σ_{yi}, and τ_i) ($i = 1, 2, \ldots, \nu$), properly chosen, can also be written in the form

$$
\begin{bmatrix}
8S_{220} & 12S_{320} & 12S_{230} & 12S_{212} & 12S_{122} \\
12S_{320} & 18S_{420} & 18S_{330} & 18S_{312} & 18S_{222} \\
12S_{230} & 18S_{330} & 18S_{240} & 18S_{222} & 18S_{132} \\
12S_{212} & 18S_{312} & 18S_{222} & 18S_{204} & 18S_{114} \\
12S_{122} & 18S_{222} & 18S_{132} & 18S_{114} & 18S_{024}
\end{bmatrix}
\times
\begin{bmatrix}
F_{12} \\ F_{112} \\ F_{122} \\ F_{166} \\ F_{266}
\end{bmatrix}
=
\begin{bmatrix}
-4 \sum_{i=1}^{\nu} \sigma_{xi}\sigma_{yi}A_i \\
-6 \sum_{i=1}^{\nu} \sigma_{xi}^2\sigma_{yi}A_i \\
-6 \sum_{i=1}^{\nu} \sigma_{xi}\sigma_{yi}^2A_i \\
-6 \sum_{i=1}^{\nu} \sigma_{xi}\tau_i^2A_i \\
-6 \sum_{i=1}^{\nu} \sigma_{yi}\tau_i^2A_i
\end{bmatrix}
\qquad (10)
$$

where

$$S_{jkl} = \sum_{i=1}^{\nu} \sigma_{xi}^j\sigma_{yi}^k\tau_i^l, \quad (j, k, l = 0, 1, 2, 3, 4)$$

$$A_i = F_1\sigma_{xi} + F_2\sigma_{yi} + F_{11}\sigma_{xi}^2 + F_{22}\sigma_{yi}^2 + F_{66}\tau_i^2 - 1$$

The surface corresponding to these values F_{12}, F_{112}, F_{122}, F_{166}, and F_{266} should be checked for its closed form. The surface is closed if the total Gaussian curvature K for the failure surface

$$K = -\frac{1}{(\partial f/\partial\sigma_x)^2 + (\partial f/\partial\sigma_y)^2 + (\partial f/\partial\tau)^2} D > 0 \qquad (11)$$

is positive (Stoker 1969; Mishchenko et al. 1985), or as the denominator is always positive, if

$$D = \begin{vmatrix}
\partial^2 f/\partial\sigma_x^2 & \partial^2 f/\partial\sigma_x\partial\sigma_y & \partial^2 f/\partial\sigma_x\partial\tau & \partial f/\partial\sigma_x \\
\partial^2 f/\partial\sigma_x\partial\sigma_y & \partial^2 f/\partial\sigma_y^2 & \partial^2 f/\partial\sigma_y\partial\tau & \partial f/\partial\sigma_y \\
\partial^2 f/\partial\sigma_x\partial\tau & \partial^2 f/\partial\sigma_y\partial\tau & \partial^2 f/\partial\tau^2 & \partial f/\partial\tau \\
\partial f/\partial\sigma_x & \partial f/\partial\sigma_y & \partial f/\partial\tau & 0
\end{vmatrix} < 0 \qquad (12)$$

If this condition is not fulfilled (i.e., the solution does not correspond to a closed failure surface), the areas of local minimum extremes have to be used. The limits of these areas are determined through a parametric investigation for any one of these five constants (e.g., for constant F_{12}). Using various values for F_{12} ($-\infty \leq F_{12} \leq +\infty$), the equivalent values for the other four constants are calculated. In this case, after elimination (10) takes the form

$$
\begin{bmatrix} 18S_{420} & 18S_{330} & 18S_{312} & 18S_{222} \\ 18S_{330} & 18S_{240} & 18S_{222} & 18S_{132} \\ 18S_{312} & 18S_{222} & 18S_{204} & 18S_{114} \\ 18S_{222} & 18S_{132} & 18S_{114} & 18S_{024} \end{bmatrix} \times \begin{bmatrix} F_{112} \\ F_{122} \\ F_{166} \\ F_{266} \end{bmatrix}
$$

$$
= \begin{bmatrix} -6 \sum_{i=1}^{\nu} \sigma_{xi}^2 \sigma_{yi} B_i \\ -6 \sum_{i=1}^{\nu} \sigma_{xi} \sigma_{yi}^2 B_i \\ -6 \sum_{i=1}^{\nu} \sigma_{xi} \tau_i^2 B_i \\ -6 \sum_{i=1}^{\nu} \sigma_{yi} \tau_i^2 B_i \end{bmatrix} \qquad (13)
$$

where

$$
B_i = F_1\sigma_{xi} + F_2\sigma_{yi} + F_{11}\sigma_{xi}^2 + F_{22}\sigma_{yi}^2 + F_{66}\tau_i^2 + 2F_{12}\sigma_{xi}\sigma_{yi} - 1
$$

Although various values of (9) are calculated, verification of (12) is also checked for each step. This verification leads to the determination of the limits inside which a closed failure surface is secured. The five values of the constants F_{12}, F_{112}, F_{122}, F_{166}, and F_{266}, which fulfill the requirement of the closed failure surface and at the same time minimize the value of (9), are selected as the solution of the problem.

APPLICATION

In the example presented, the method has been applied through a specific computer program developed by the writers in FORTRAN programming language. With this program, the failure surface is determined for a real case of a masonry material already studied experimentally (Page 1981). These data have been used by many other researchers [e.g., Dhanaseker et al. (1985) and Naraine and Sinha (1991)].

The experimental values for the monoaxial failure strength estimated from graphs (Page 1981) are taken as equal to X = 0.40 MPa, X' = 4.3625 MPa, Y = 0.10 MPa, Y' = 7.555 MPa, and $S = S'$ = 0.40 MPa. Using these values as well as (4)–(7), the constants F_i and F_{ii} are determined with the values F_1 = 0.227E+01 (MPa)$^{-1}$, F_{11} = 0.573E+00 (MPa)$^{-2}$, F_2 = 0.987E+01 (MPa)$^{-1}$, F_{22} = 0.132E+01 (MPa)$^{-2}$, and F_{66} = 0.625E+01 (MPa)$^{-2}$.

To determine the constants F_{12}, F_{112}, F_{122}, F_{166}, and F_{266}, (10) is solved using experimental data of Tables 1 and 2. The results corresponding to these data are F_{12} = 1.640209E−02 (MPa)$^{-2}$, F_{112} = 9.453948E−03 (MPa)$^{-3}$, F_{122} = 8.886827E−03 (MPa)$^{-3}$, F_{166} = 1.365468E−01 (MPa)$^{-3}$, and F_{266} = 1.2929976E−01 (MPa)$^{-3}$.

The surface corresponding to these values should be checked for its closed form. The surface (Fig. 4) is opened because its total curvature is negative [(12) is not valid in this case].

TABLE 1. Data of Biaxial Tests

Test number (1)	σ_x (MPa) (2)	σ_y (MPa) (3)	τ (MPa) (4)
1	−0.727	−7.542	0.000
2	−0.727	−8.417	0.000
3	−2.272	−9.250	0.000
4	−2.181	−8.750	0.000
5	−4.545	−8.667	0.000
6	−7.909	−7.791	0.000
7	−8.818	−8.750	0.000
8	−9.454	−4.792	0.000
9	−9.590	−2.333	0.000
10	−11.273	−5.583	0.000
11	−9.272	−1.000	0.000

Note: These values have been estimated from graphs (Page 1981).

TABLE 2. Data of Biaxial Tests

Test number (1)	σ_x (MPa) (2)	σ_y (MPa) (3)	τ (MPa) (4)
1	−4.181	−8.000	0.000
2	−9.909	−5.042	0.000
3	−8.308	−8.475	0.084
4	−4.555	−1.310	−1.622
5	−5.821	−5.821	3.571
6	−6.620	−6.620	2.120
7	−5.821	−5.821	−3.571
8	−6.620	−6.620	−2.120
9	−8.273	−8.475	−0.084
10	−5.227	−1.310	1.622
11	−4.181	−8.000	0.000
12	−9.909	−5.042	0.000

Note: These values have been estimated from graphs (Page 1981).

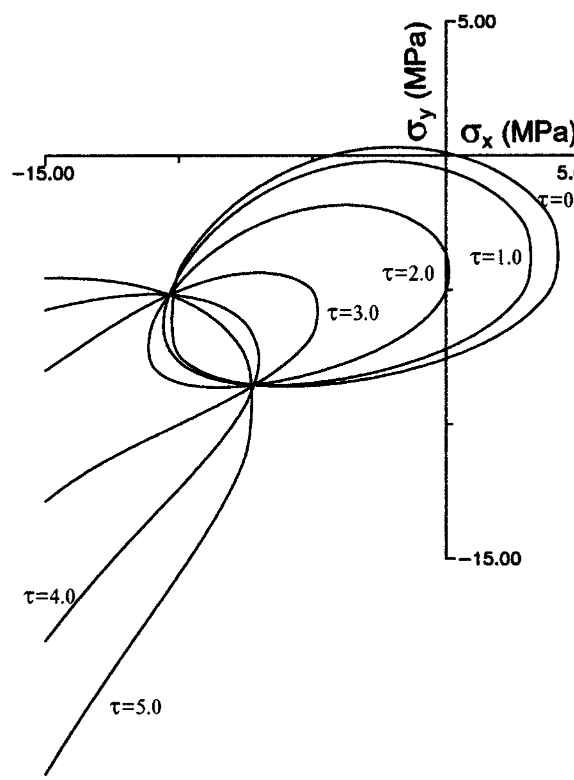

FIG. 4. Open Failure Surface of Masonry in Terms of Normal Stress (τ = 0.0, 1.0, 2.0, 3.0, 4.0, and 5.0 MPa)

As the solution achieved does not correspond to a close failure surface, the areas of local minimum extremes have to be used. The limits of these areas are determined through a parametric investigation for any one of the five constants (e.g., for constant F_{12}).

For various values of F_{12} ($-\infty \le F_{12} \le +\infty$), the equivalent values for the other constants are calculated. Various values of (9) are calculated (Fig. 5), and verification of (12) is checked for each step. This verification leads to the determination of the limits inside which a closed failure surface is secured. The five values of the constants F_{12}, F_{112}, F_{122}, F_{166}, and F_{266} fulfilling the requirement of the closed failure surface and at the same time minimizing the value of (9) are chosen as the solution of the problem.

The area in which the closed failure surface is ensured as shown in Fig. 5. This area is

$$
-0.710 \le F_{12} \le -0.150
$$

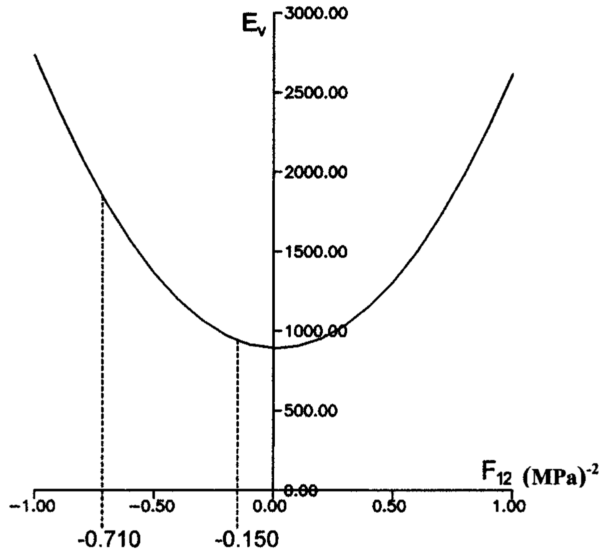

FIG. 5. Variation of E_v with F_{12}, Following Eq. 9

According to this diagram, the best solution corresponds to the value of $F_{12} = -0.150$ (MPa)$^{-2}$. For this value, the equivalent values for the other constants are calculated equal to $F_{112} = 0.3195E{-}02$ (MPa)$^{-3}$, $F_{122} = 0.1045E{-}02$ (MPa)$^{-3}$, $F_{166} = 0.9466E{-}01$ (MPa)$^{-3}$, and $F_{266} = 0.1563E{+}00$ (MPa)$^{-3}$.

The failure surface (Fig. 6) for the masonry is described in the equation

$$2.27\sigma_x + 9.87\sigma_y + 0.573\sigma_x^2 + 1.32\sigma_y^2 + 6.25\tau^2 - 0.30\sigma_x\sigma_y + 0.009585\sigma_x^2\sigma_y + 0.003135\sigma_x\sigma_y^2 + 0.28398\sigma_x\tau^2 + 0.4689\sigma_y\tau^2 = 1 \quad (14)$$

The validity of the method is demonstrated by comparing the derived analytical failure surface of (14) with the existing experimental results (Page 1981). More than 100 experimental data have been depicted in Figs. 7–9. In the same figures, analytical curves are also depicted for the failure surface of (14). The good coincidence of the experimental and analytical data is obvious for this general failure surface with a nonsymmetric curve.

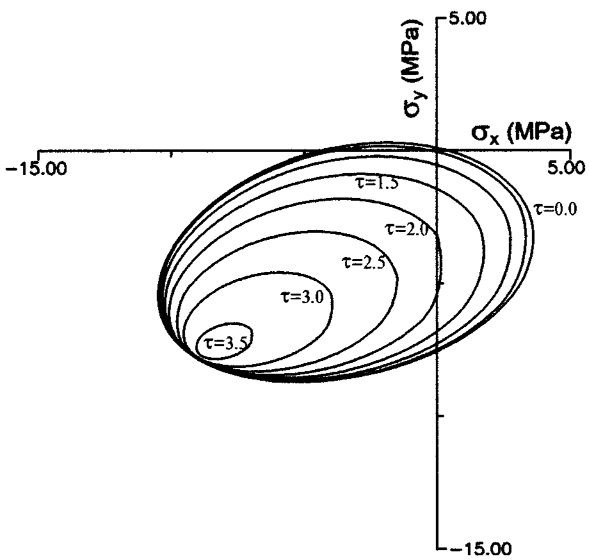

FIG. 6. Failure Surface of Masonry in Normal Stress Terms (τ = 0.0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, and 3.5 MPa)

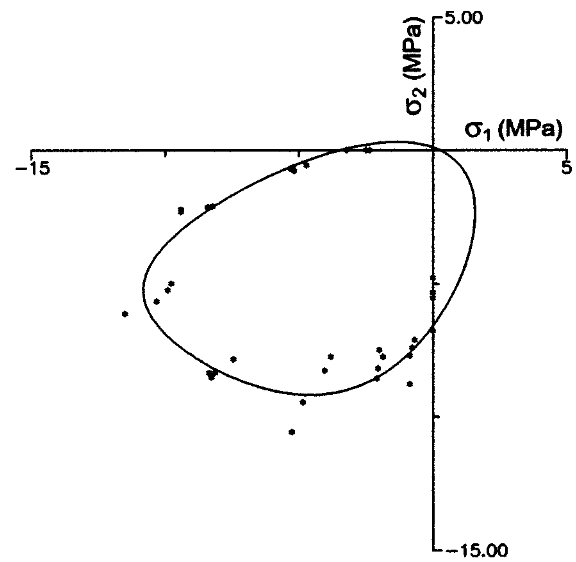

FIG. 8. Failure Curve of Masonry in Principal Stress Terms (θ = 22.5°)

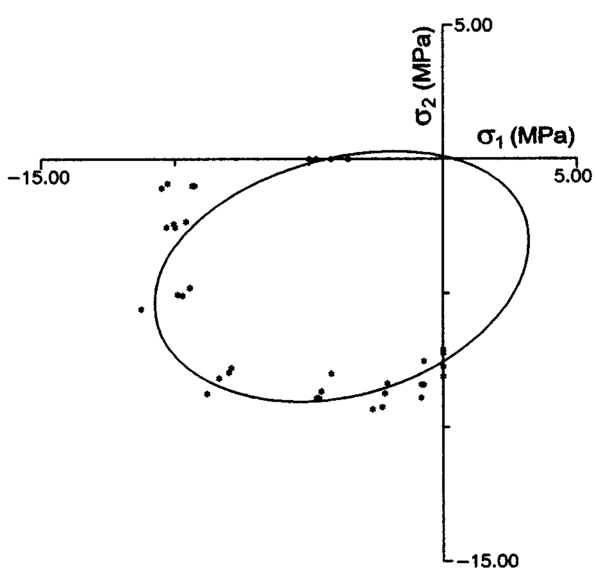

FIG. 7. Failure Curve of Masonry in Principal Stress Terms (θ = 0°)

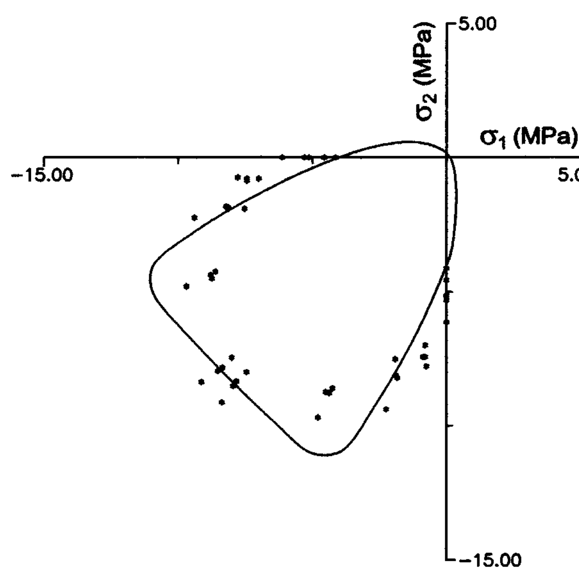

FIG. 9. Failure Curve of Masonry in Principal Stress Terms (θ = 45°)

COMPARISON TO SIMPLIFIED MODEL

Eliminating all third-order terms in (3), a simplified failure criterion is derived

$$f(\sigma_x, \sigma_y, \tau) = F_1\sigma_x + F_2\sigma_y + F_{11}\sigma_x^2 + F_{22}\sigma_y^2 + F_{66}\tau^2 + 2F_{12}\sigma_x\sigma_y - 1 = 0 \quad (15)$$

This simple form of the criterion has already been used by Dhanasekar et al. (1985), Scarpas (1991), Andreaus (1996), and Syrmakezis and Asteris (1999).

The constants F_i and F_{ii}, calculated through monoaxial test results, have the same values as for the general form of the failure criterion. Using the data of Tables 1 and 2, the constant F_{12} is calculated through the formula (Syrmakezis and Asteris 1999)

$$F_{12} = -\frac{\sum_{i=1}^{\nu} (F_1\sigma_{xi} + F_2\sigma_{yi} + F_{11}\sigma_{xi}^2 + F_{22}\sigma_{yi}^2 + F_{66}\tau_i^2 - 1)\sigma_{xi}\sigma_{yi}}{2\sum_{i=1}^{\nu} \sigma_{xi}^2\sigma_{yi}^2} \quad (16)$$

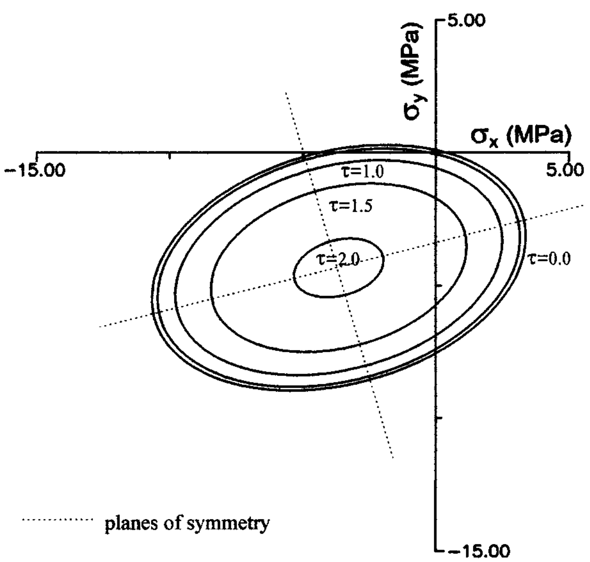

FIG. 10. Failure Surface of Masonry in Normal Stress Terms (τ = 0.0, 0.5, 1.0, 1.5, and 2.0 MPa)

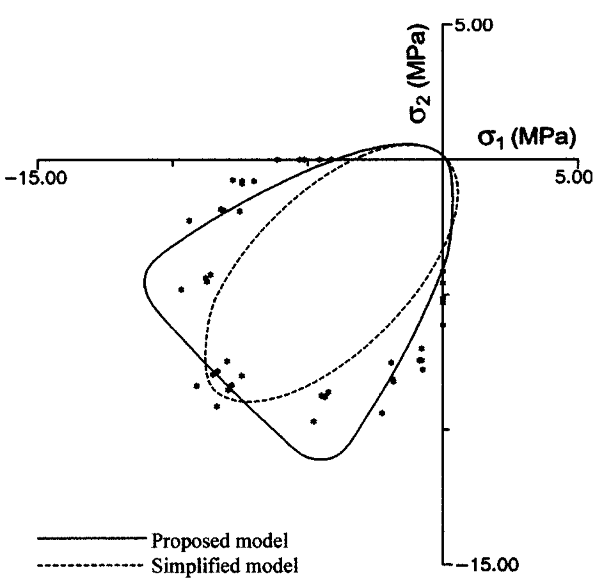

FIG. 11. Failure Curve of Masonry in Principal Stress Terms (θ = 45°)

Using this formula, the constant F_{12} is calculated equal to −0.227 (Syrmakezis and Asteris 1999) and the failure surface for this simplified model has an elliptical shape, as shown in Fig 10.

In Fig. 11, the simplified model (dotted line) is compared with the general model (continuous line) for the case of an angle θ (angle between the maximum principal stress direction and the direction of the x-axis) equal to 45°. It is to be noted that the general failure criterion, through its nonsymmetric form, can approach the nonsymmetrically dispersed experimental data better than the simplified model.

CONCLUSIONS

In this paper a method is presented for the analytical determination of the failure surface of an anisotropic (orthotropic) masonry under biaxial stress. The main advantages of the method can be summarized as follows:

- The ability to ensure the closed shape of the failure surface
- The unique mathematical form for all possible combinations of plane stress to make it easier to include it into existing software for the analysis of masonry structures
- The satisfactory approximation with the results of the real masonry behavior (experimental data) under failure conditions

APPENDIX I. REFERENCES

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APPENDIX II. NOTATION

The following symbols are used in this paper:

F_i = strength tensor second rank;
F_{ij} = strength tensor fourth rank;
F_{ijk} = strength tensor sixth rank;
S, S' = positive and negative pure shear strengths, respectively;
X, X' = tensile and compressive strengths measured along *x*-axis, respectively;
Y, Y' = tensile and compressive strengths measured along *y*-axis, respectively;
θ = angle between maximum principal stress direction and direction of *x*-axis;
σ_x, σ_y = normal plane stresses along *x*-axis and *y*-axis, respectively;
σ_1, σ_2 = maximum and minimum principal stresses, respectively; and
τ = shear stress measured in *x*, *y*-plane.