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NONLINEAR ANALYSIS USING REGULAR YIELD SURFACE

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ABSTRACT

In this paper, a new methodology is presented to non-linear analysis of masonry shear wall under biaxial stress state using the finite element method. The methodology focuses on the definition / specification of the yield surface for the case of anisotropic masonry under biaxial stress state, as well as on the numerical solution of this non-linear finite elements problem. Specifically, in order to define the yield surface we use a cubic tensor polynomial, whereas we use the initial stress method in order to solve the elasto-plastic problem. In addition, novel computer code of finite elements has been developed in order to apply the method of elasto-plastic analysis of plane masonry wall. The main advantages of the method can be summarized as follows: a) The plasticity equations through a regular surface leads to the elimination of the problem that occurs by the use of singular surface, and b) It is clearly shown that the non-linear behavior of masonry is strongly affected by the yield criterion.

Keywords: Computer code, masonry, non-linear analysis, regular yield surface, shear wall

INTRODUCTION

In the present work we outline the basic assumptions and associated mathematical expressions for a theory of plasticity, giving special attention in the case of masonry. More specifically, in order to formulate the quantitative expressions of the mathematical theory of plasticity, a new analytic method has been used for the description of the yield of the anisotropic masonry via a **regular** surface, that is, a surface defined by a single equation of the form $f(\sigma) = 0$ (Koiter, 1953).

The significance of the use of a regular yield surface has been manifested since 1950, when Hill introduced in his book "The Mathematical Theory of Plasticity". The theory of plasticity through a closed yield surface encounters the existence of singular points on the yield surface. This problem imposes additional computational difficulties in the non-linear analysis procedure (Zienkiewcz, Valliapan and King, 1969).

An additional problem of the up-to-day non-linear behavior analysis of masonry (Andreaus & Ippoliti, 1995; Ballio *et al*., 1992) is the use of ready made analysis software packages that have been developed for the case of concrete. The basic disadvantage of these ready-made software packages is that their architecture is not amenable to modifications that take into account some important assumptions, which are valid for the case of masonry.

To overcome these problems, a novel computer code, in FORTRAN programming language, has been developed. The code can be applied in elasto-plastic anisotropic masonry wall under

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plane stress. Special attention has been given, during the development procedure, in graphic imaging of the analysis results. The software has also the capability of automatically producing not only the load – displacement diagram, but also the graphic images of the yield process, which are colored according to the kind of stress under which yield takes places.

THE MATHEMATICAL THEORY OF PLASTICITY

In order to formulate a theoretical description, capable to model elasto-plastic material deformation, three requirements have to be addressed:

- An explicit relationship between stress and strain that will describe the material's behavior under elastic conditions.
- A yield criterion that will define the stress level at which plastic flow commences must be postulated.
- A relationship between stress and strain must be developed for post-yield behaviour, i.e. when the deformation is made up of both elastic and plastic components.

The relationship between stress and strain before the onset of plastic yielding is given by the following standard linear elastic expression:

$$\sigma = D\varepsilon \qquad (1)$$

In this expression ó and å are the stress and strain components, respectively, and D is the elasticity matrix.

The yield criterion

The yield criterion defines the stress level at which plastic deformation begins and takes the form of the equation:

$$f(\sigma) = 0 \qquad (2)$$

where f is a function.

The geometry of the yield surface tends to have a significant influence not only in the formulation, but also in the numerical solution of the non-linear problem, as we will show in the next paragraph where we will present in detail all the problems relevant to the estimation of elasto-plastic matrix.

Plastic flow Rule

Von Mises first suggested, in 1928, the basic constitutive relation that defines the plastic strain increments in relation to the yield surface. Various other researchers (Drucker, 1951; Prager, 1956) have proposed heuristic methods for the validation of Von Mises proposed relationship. These methods have led to the current state-of-the-art hypothesis, which states that:

If $\delta\{\varepsilon\}_p$ denotes the increment of plastic strain, then:

$$\delta\{\varepsilon\}_p = \lambda \frac{\partial f}{\partial\{\sigma\}} \qquad (3)$$

where λ is a determinable constant (plastic multiplier).

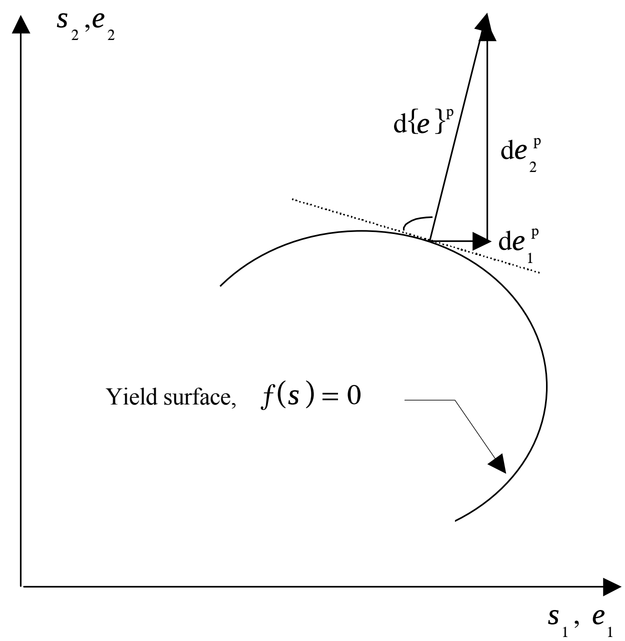

FIG. 1. Geometrical representation of the normality rule in 2D Stress Space

This rule is widely known as the normality principle because the relation (3) can be interpreted as requiring the normality of the plastic strain increment vector to the yield surface in the hyper-space of í stress dimensions. In Fig. 1 this normality rule is shown in the case of a two dimensional space.

Stress–strain relations

During an infinitesimal increment of stress, changes of strain are assumed to be partly elastic and partly plastic as:

$$\delta\{\varepsilon\} = \delta\{\varepsilon\}_e + \delta\{\varepsilon\}_p \tag{4}$$

The elastic strain increments are related to the stress increments via a symmetric matrix of constants [D] known as the elasticity matrix:

$$\delta\{\varepsilon\}_e = [D]^{-1}\delta\{\sigma\} \tag{5}$$

Expression (4) can be readily rewritten as

$$\delta\{\varepsilon\} = [D]^{-1}\delta\{\sigma\} + \frac{\partial f}{\partial\{\sigma\}}\lambda \tag{6}$$

Manipulation of the above equations leads to the following elasto-plastic stress-strain relation:

$$\delta\{\sigma\} = D_{ep}\delta\{\varepsilon\} \tag{7}$$

where:

$$D_{ep} = D - \frac{D\alpha\alpha^T D}{\alpha^T D\alpha} \tag{8}$$

is the elasto-plastic matrix, and

$$\alpha = \left\{\frac{\partial f}{\partial\{\sigma\}}\right\} \tag{9}$$

is the flow vector.

YIELD SURFACE GEOMETRY EFFECT IN NON-LINEAR SOLUTION

As we have already mentioned in the previous paragraph, the yield criterion affects the formulation of the non-linear problem. In this paragraph we will describe the yield surface geometry effect in the formulation and the numerical solution of the elasto-plastic problem.

"Corners" in a yield surface

Sometimes the yield surface is not defined by only a single continuous (and convex) function, but by a series of functions.

According to Koiter (1953), a surface of this kind is called singular. Such a surface is the yield surface of Tresca and the surface about the masonry in three mutually intersected cones proposed by Dhanasekar, Page, and Kleeman (1985).

According to Zienkiewicz, Valliapan, and King (1969), the use of singular areas imposes important problems to the elasto-plastic analysis process. The researchers propose to avoid calculating the singular points in yield surface by a suitable choice of continuous surfaces, which usually can with a good degree of accuracy represent the true conditions.

Regular yield surface

Having in mind the computational problems introduced during the formulation and the numerical solution of the non linear problem using singular yield surfaces, we will use a new method in order to define the yielding for the case of anisotropic masonry (Syrmakezis and Asteris, 2001) via a **regular** surface, that is, a surface defined by a single equation of the form:

$$\begin{aligned} f(\sigma_x, \sigma_y, \tau) = {} & F_1\sigma_x + F_2\sigma_y + F_{11}\sigma_x^2 + F_{22}\sigma_y^2 + F_{66}\tau^2 + 2F_{12}\sigma_x\sigma_y + 3F_{112}\sigma_x^2\sigma_y + \\ & + 3F_{122}\sigma_x\sigma_y^2 + 3F_{166}\sigma_x\tau^2 + 3F_{266}\sigma_y\tau^2 - 1 = 0 \end{aligned} \tag{10}$$

Eliminating all third order terms in eq. 10, a simplified yield criterion is derived:

$$f(\sigma_x, \sigma_y, \tau) = F_1\sigma_x + F_2\sigma_y + F_{11}\sigma_x^2 + F_{22}\sigma_y^2 + F_{66}\tau^2 + 2F_{12}\sigma_x\sigma_y - 1 = 0 \tag{11}$$

This latter simple form of the criterion has already been used by Dhanasekar, Page and Kleeman (1985) and Andreaus (1996).

According to Syrmakezis and Asteris (2001) the general yield criterion (10) through its non-symmetric form, fit the non-symmetrically dispersed experimental data better than the simplified model (11).

COMPUTER CODE

In order to implement the method, a specific computer program for a 2D non-linear finite element analysis of masonry plane wall under monotonic static loads has been developed. During the development procedure, we have made use of the ready-made databanks of Owen & Hinton PLAST computer code (Owen & Hinton, 1982)

It must be mentioned that many other researchers have used Owen & Hinton software in order to develop non-linear software for the analysis of masonry. The most representative is a non-linear analysis computer code developed by Adreaus (1996). The main disadvantages of the Owen & Hinton software are the isotropic consideration of the materials and the use of isotropic yield criteria.

For the non-linear solution the method of initial stiffness proposed by Zienkiewicz, Valliapan and King (1969) has been used. According to authors this method can calculate an elasto-plastic problem based on a series of successive approximations.

The software used in the present research, overcomes the above mentioned disadvantages of PLAST, and is appropriate to model the anisotropic behavior of the materials, allowing the use of anisotropic and regular yield surface (Eqs. 10 and 11). During the development phase we gave special attention to the graphic representation of the analysis results. Also, with this software we can produce not only the load–displacement diagram, but also the graphic images of the yield process colored according to the kind of stress (yield under biaxial compressive, tensile or heterosemous stress).

APPLICATION

Using this computer program, we studied the non-linear behavior of a plane masonry wall with openings under uniform compressive and shear loading as shown in Fig. 2 with the following assumptions:

- The loads are uniformly distributed at the wall top; the reference load amplitude in both directions is assumed to equal 0.1 MPa and the load factor increment is equal to 0.1.
- The masonry wall has been discretized by means of four-node isoparametric quadrilateral elements, whose length is 1.00 m.
- Isotropic linearly elastic behaviour has been assumed for masonry material in the purely elastic range, with Young's modulus E=5700 MPa and Poisson's ratio ν=0.19.
- The use of both a simple and a general yield criterion.

For the mechanical characteristics of the masonry, we used the experimental results of Page (1981). With the same results they defined (Asteris, 2000; Syrmakezis and Asteris, 2001) the regular yield surface for the case of the general yield criterion (Eq. 10) through the equation:

$$\begin{aligned}&2.27\sigma_x + 9.87\sigma_y + 0.573\sigma_x^2 + 1.32\sigma_y^2 + 6.25\tau^2 - 0.30\sigma_x\sigma_y + 0.009585\sigma_x^2\sigma_y + 0.003135\sigma_x\sigma_y^2 + \\ &+ 0.28398\sigma_x\tau^2 + 0.4689\sigma_y\tau^2 = 1\end{aligned} \quad (12)$$

as well as for the case of the simplified yield criterion (Eq. 11)) through the equation:

$$2.27\sigma_x + 9.87\sigma_y + 0.573\sigma_x^2 + 1.32\sigma_y^2 + 6.25\tau^2 - 0.454\sigma_x\sigma_y = 1 \quad (13)$$

The regular yield surface described by Eq. (12) is depicted in Figure 3a, whereas the simplified yield surface described by Eq. (13) is depicted in Figure 3b.

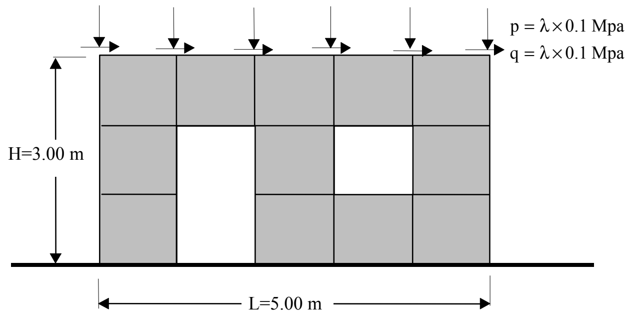

FIG. 2. Shear wall with openings under uniform compressive and shear loading

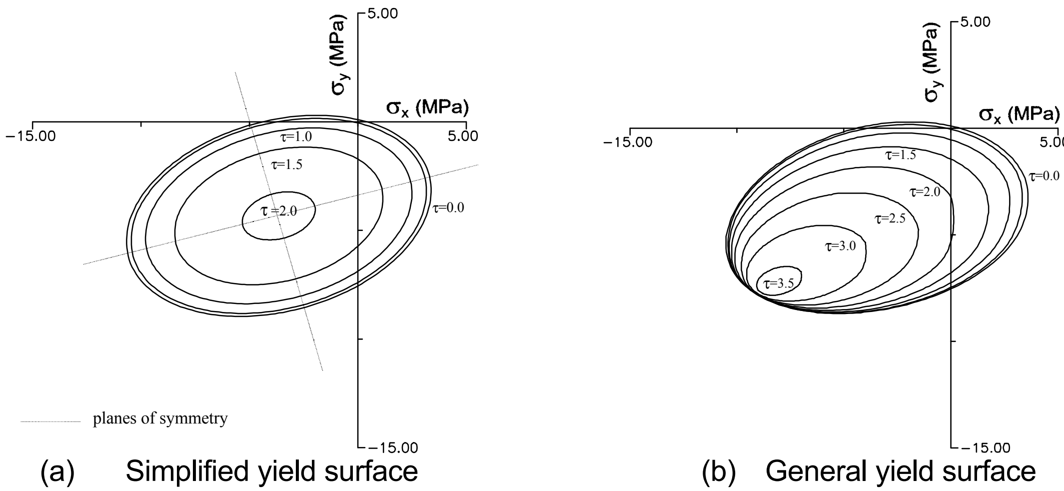

FIG. 3.Yield surface of masonry in normal stress terms (Asteris, 2000; Syrmakezis and Asteris; 2001)

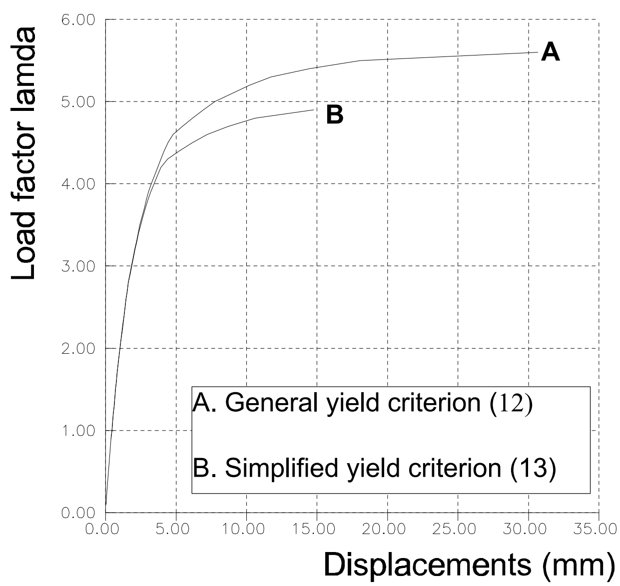

FIG. 4. Load factor lamda (λ)-displacement diagram

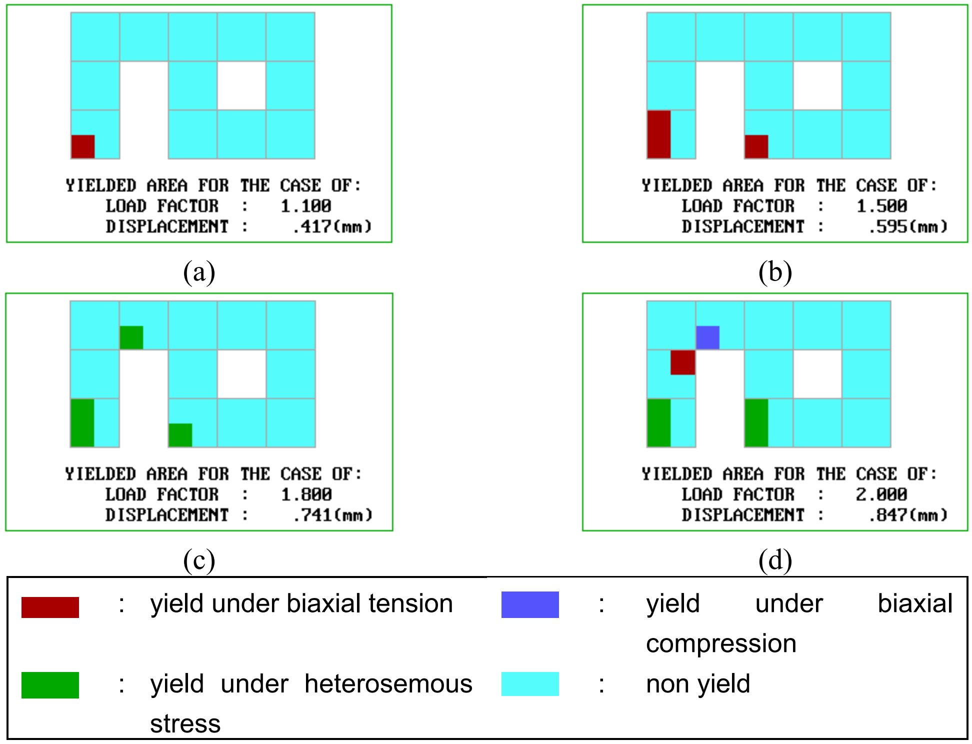

FIG. 5. Successive representations of yield pattern

Fig. 4 shows the load factor λ – displacement diagram using both a simple and a general yield. It is clear that the non-linear behavior of masonry is affected by the yield criterion. It must be noted that this strong variation appeared during the study of this wall although both criteria have the same mechanical masonry characteristics (same mono-axial compressive and tensile strength as well as the same strength in pure shear).

With this software we can produce not only the load–displacement diagram (Fig. 4), but also the graphic images of the yield process (Fig. 5), which are colored according to the kind of stress (yield under biaxial compressive, tensile or heterosemous stress). These graphic representations are especially useful not only because of the information they give, but also because of the validation they provide.

CONCLUSIONS

The present research shows a new methodology for the non-linear 2D finite element analysis of anisotropic masonry under monotonic loads. The methodology focuses on the definition / specification of the yield surface for the case of anisotropic masonry under biaxial stress state, as well as on the numerical solution of this non-linear problem. Specifically, in order to define the yield surface we use a cubic tensor polynomial, whereas we use the initial stress method in order to solve the elasto-plastic problem. In addition, novel computer code of finite elements has been developed in order to apply the method of elasto-plastic analysis of plane masonry wall, which takes account of their specifically intense anisotropic behavior. The main advantages of the method can be summarized as follows: a) The plasticity equations through a regular surface leads to the elimination of the problem that occurs by the use of singular surface, and b) It is clearly shown that the non-linear behavior of masonry is strongly affected by the yield criterion.

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APPENDIX I. NOTATION

The following symbols are used in this paper:

D_{ep} = elasto-plastic matrix;

E = Young's modulus;

F_i = strength tensor second rank;

F_{ij} = strength tensor fourth rank;

F_{ijk} = strength tensor sixth rank;

α = flow vector;

λ = both plastic multiplier and load factor;

ν Poisson's ratio;

σ_x, σ_y = normal plane stresses along x-axis and y-axis, respectively;

τ = shear stress measured in the x, y- plane;