Strength of Unreinforced Masonry Walls under Concentrated Compression Loads

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Abstract: In this paper, the strength of masonry walls subjected to concentrated vertical loads is investigated. An orthotropic finite-element model is used to simulate the wall behavior. Nonlinear deformation characteristics of masonry material as well as its anisotropic (orthotropic) behavior are taken into consideration. A new anisotropic yield/failure surface of a masonry wall under biaxial stress, in a cubic tensor polynomial form, is proposed. A parametric study is carried out using several parameters such as the loaded area length to the total length ratio, the load position in relation to the end of the wall, and the wall geometry. Based on the results of the parametric investigation, a new design rule that takes into account the aforementioned parameters is proposed.

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Introduction

When a wall is subjected to a concentrated load, the compressive resistance of the masonry in the area immediately under the load is often greater than the normal compressive strength of the wall. This phenomenon occurs due to the apparent local strengthening of the material beneath the load, which restrains its lateral expansion (Poisson's effect). The magnitude of this strength enhancement depends upon the loaded area ratio, the wall geometry, and the location of the load along the wall, with the degree of enhancement reduced as the load approaches the end of the wall. Although some form of stress enhancement factor is given in most masonry design codes, its nature and magnitude varies considerably from country to country.

One of the difficulties in predicting the behavior of masonry walls subjected to concentrated loads is the lack of realistic stress analysis of the masonry, because the region beneath the load is in a state of biaxial stress with high stress gradients. The in-plane deformation and failure of masonry is influenced by the properties of its components, the units and the mortar. The influence of the mortar joints is particularly significant, as these joints act as weak planes. The results of previous studies of concentrated loading on more isotropic homogeneous material (such as concrete) are thus not directly applicable. To model the behavior of masonry walls subjected to concentrated loads, a more representative material model is required.

This paper describes an analytical investigation of the behavior of masonry walls subjected to concentrated loads, using a finite-element program for nonlinear anisotropic analysis. A total of 62 walls are analyzed, with the effect of loaded area ratio, β $(=a'/a)$, the load location in relation to the end of the wall, d/a, and the aspect ratio of the wall, α $(=a/h)$ (as defined in Fig. 1) being investigated. From the results, a design rule incorporating all of the preceding variables is proposed and compared to the existing design rules from various masonry codes.

Literature Review

On account of the lack of comprehensive research in this area, existing design rules are semiempirical and vary considerably from country to country. The large number of variables that must be considered has hampered experimental investigations. Analytical studies have usually assumed isotropic elastic behavior, thus ignoring the effect of material nonlinearity. Existing rules do not take account of the aspect ratio of the wall, and most of them do not consider the location of the load along the wall.

Existing design rules for predicting the capacity of walls subjected to concentrated loads reflect the empirical nature of the provisions. Increased stresses are typically allowed immediately beneath the concentrated load, usually by the use of a stress enhancement factor, which is a function of the loaded are ratio. The most characteristic design rules for the strength enhancement factor γ are the following:

- EC6 (Comite European de Normalisation 1988):

$$\gamma = 1.0 + 0.1\frac{d}{a'} \leq 1.50 \tag{1}$$

- Chinese code GBJ-3-73 (Dia-Xin 1985)

$$\gamma = \sqrt{\frac{1}{\beta} - 1} \tag{2}$$

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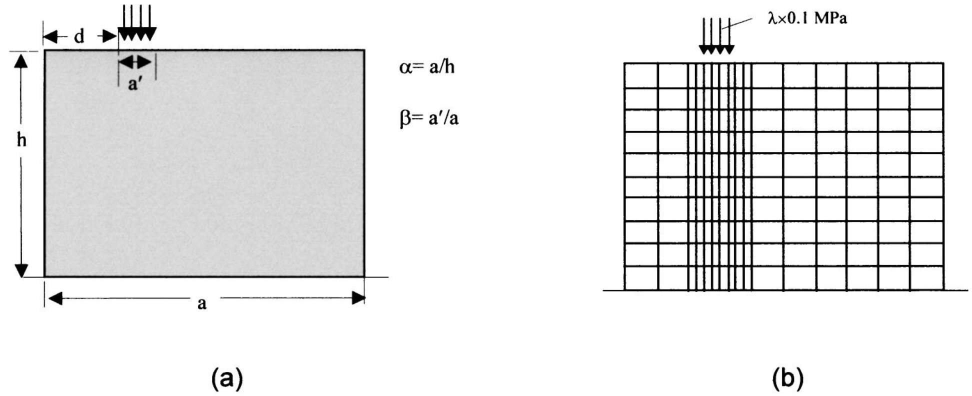

Fig. 1. Concentrated load on a wall: (a) Geometry and terminology; and (b) finite-element mesh

- Dai-Xin (1985) for concentric load

$$\gamma = 1 + 0.708\sqrt{\frac{1}{\beta} - 1} \leqslant 3.00 \qquad (3)$$

and for eccentric load

$$\gamma = 1 + 0.364\sqrt{\frac{1}{\beta} - 1} \leqslant 2.00 \qquad (4)$$

- Ali and Page (1988a,b)

$$\gamma = \frac{[0.80 + (1-\beta)d/a](2 + 1/\beta)^{1/2}}{1.80\dfrac{1 + 0.40\alpha}{1 + 0.20\alpha}} \geqslant 1.00 \qquad (5)$$

- Malek and Hendry (1988) for central load

$$\gamma = 0.701\beta^{-0.462} \qquad (6)$$

for eccentric load

$$\gamma = 0.806\beta^{-0.393} \qquad (7)$$

and for end load

$$\gamma = 0.856\beta^{-0.266} \qquad (8)$$

One should mention that the equations proposed by Ali and Page (1988a,b) as well as by Chinese code GBJ-3-73 (Dai-Xin 1985) overestimate the strength of the wall as compared to the Eurocode equation. In addition, all the aforementioned equations limit themselves to yield values greater than 1 (>1) of the strength enhancement factor γ. However, for the case of eccentric load, values less than 1 (<1) of the factor γ have been measured experimentally by Arora (1988).

Method of Analysis

Analytical and experimental studies on the behavior of masonry walls subjected to in-plane static loads have been the focus of activity of a number of investigators for many years. Masonry exhibits distinct directional properties, due to the influence of mortar joints acting as planes of weakness. Depending upon the orientation of the joints to the stress directions, failure can occur in the joints alone, or simultaneously in the joints and blocks. The great number of influencing factors, such as the dimension and anisotropy of the bricks, the joint width and arrangement of bed and head joints, the material properties of both the brick and mortar, and the quality of workmanship, make the simulation of plain brick masonry extremely difficult.

The failure of masonry under uniaxial and biaxial stress states has been studied extensively in the past. All these cases of failure represent particular points on the general failure surface. The development of a general yield criterion for masonry is difficult because of the difficulties in developing a representative biaxial test and the large number of tests involved.

In the absence of a suitable model to represent its behavior, it the past, masonry was assumed to be an isotropic elastic continuum; consequently, the influence of the mortar joints acting as planes of weakness could not be addressed. The development of improved models of material behavior was made possible by the increased sophistication of numerical methods of stress analysis. Indeed, only recently have analytical procedures that account for the nonlinear behavior of masonry under static loads been developed. These analytical procedures could be summarized in the following two levels of refinement for masonry models:

1. ***Macromodeling (masonry as a one-phase material)***. According to this procedure (Andreaus 1996; Asteris 2000), no distinction between the individual units and joints is made, and masonry is considered as a homogeneous, isotropic, or anisotropic continuum. While this procedure may be preferred for the analysis of large masonry structures, it is not suitable for the detailed stress analysis of a small panel, due to the fact that is difficult to capture all its failure mechanisms. The influence of the mortar joints acting as planes of weakness cannot be addressed.
2. ***Micromodeling (masonry as a multiphase material)***. According to this procedure (Page 1978; Ali and Page 1988a,b; Lourenco and Ross 1993, 1994; Lourenco 1996), the units, the mortar, and the unit/mortar interface are modeled separately. While this leads to more accurate results, the level of refinement can be computationally very intensive, and thus its application is limited to small laboratory specimens and structural details. Sutcliffe et al. (2001) have recently proposed simplified micromodeling procedures to overcome the problem. According to these procedures, which are intermediate approaches, the properties of the mortar and the unit/mortar interface (masonry as a two-phase material) are lumped into a common element, while expanded elements are used to represent the brick units. This approach leads to a reduction in computational intensity and yields a model that is applicable to a wider range of structures.

In the present paper, a complete methodology for the nonlinear

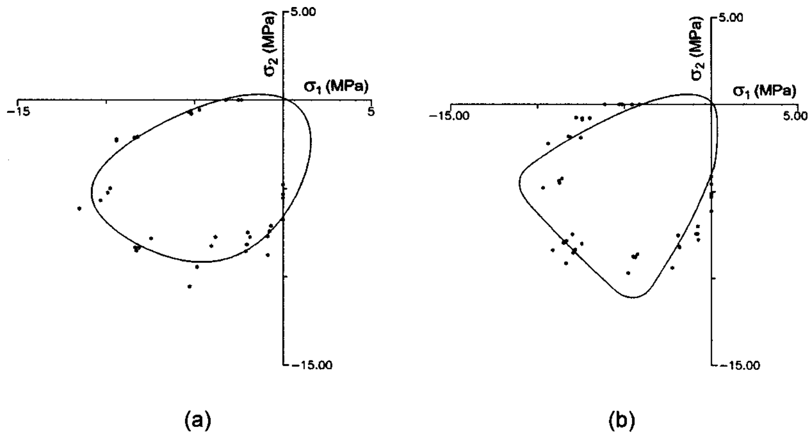

Fig. 2. Yield curves of masonry in principal stress terms: (a) $\theta = 22.5°$; and (b) $\theta = 45.00°$ (Syrmakezis and Asteris 2001)

analysis of anisotropic unreinforced masonry shear walls under concentrated compression loads is presented, regarding masonry as a one-phase material. One of the advantages of the proposed material model is that average properties that include the influence of both brick and joint have been used. This means that a relatively coarse finite-element mesh can be used, with any element typically encompassing several bricks and joints. Such an approach has considerable computational advantages when analyzing large wall panels.

The method comprises the following:

- A finite element to model the in-plane anisotropic behavior of a masonry wall panel;
- A *regular* anisotropic yield surface [according to Koiter (1953), a yield surface is called regular if it is described by a continuous function]; and
- A finite-element computer code for nonlinear analysis, which takes into account the distinct anisotropic nature of masonry.

Finite-Element Model

A four-node isoparametric rectangular finite-element model with 8 degrees of freedom (DOF) has been used for the analysis in the present paper. In order to take into account the distinct anisotropic/orthotropic behavior of masonry that exhibits directional properties due to the influence of mortar joints acting as planes of weakness, the material is assumed to be homogeneous and anisotropic. In particular, the material of the masonry shows a different modulus of elasticity, (E_x), in the x direction (direction parallel to the bed joints of masonry) and (E_y) in the y direction (perpendicular to the bed joints). In the case of plane stress, the elasticity matrix is defined by

$$\mathbf{D} = \begin{bmatrix} \dfrac{E_x}{1-\nu_{xy}\nu_{yx}} & \dfrac{E_x\nu_{yx}}{1-\nu_{xy}\nu_{yx}} & 0 \\ \dfrac{E_y\nu_{xy}}{1-\nu_{xy}\nu_{yx}} & \dfrac{E_y}{1-\nu_{xy}\nu_{yx}} & 0 \\ 0 & 0 & G_{xy} \end{bmatrix} \tag{9}$$

where ν_{xy} and ν_{yx} = Poisson's ratios in the xy and yx plane, respectively; and G_{xy} = shear modulus in the xy plane. In particular, in the case of plane stress in an anisotropic materials, the following equation holds:

$$E_x\nu_{yx} = E_y\nu_{xy} \tag{10}$$

The shear modulus of elasticity G_{xy} can be derived either experimentally or analytically, through the following expression (Page et al. 1985; Asteris 2003):

$$G_{xy} = \frac{\sqrt{E_xE_y}}{2(1+\sqrt{\nu_{xy}+\nu_{yx}})}$$

Masonry Failure Criterion under Biaxial Stress State

The failure theories for isotropic materials are not applicable for masonry under biaxial stresses because they are derived on the basis of the invariant state of stress concept, where the stress orientation has no effect on the strength. In this section, failure criteria are proposed as a generalized form for masonry under biaxial stresses, taking into consideration its anisotropic nature as a composite material. The mortar bed joints, because of their continuous nature, divide the media into layers of equal thickness and thus give masonry the appearance of a laminated composite material. For the expression of an analytical failure model of masonry, therefore, a polynomial that is available already for composite materials is proposed. This failure surface in the stress space can be described by the following equation (Asteris 2000; Syrmakezis and Asteris 2001):

$$\begin{aligned} f(\sigma_x,\sigma_y,\tau) &= F_1\sigma_x + F_2\sigma_y + F_{11}\sigma_x^2 + F_{22}\sigma_y^2 + F_{66}\tau^2 + 2F_{12}\sigma_x\sigma_y \\ &\quad + 3F_{112}\sigma_x^2\sigma_y + 3F_{122}\sigma_x\sigma_y^2 + 3F_{166}\sigma_x\tau^2 + 3F_{266}\sigma_y\tau^2 - 1 \\ &= 0 \end{aligned} \tag{11}$$

In order to determine the coefficients of the proposed cubic tensor polynomial [Eq. (11)], an evaluation of the mechanical characteristics of masonry is performed using the experimental data of Page (1981), through a least squares approach. The yield surface can then be defined (Asteris 2000; Syrmakezis and Asteris 2001) as

$$\begin{aligned} &2.27\sigma_x + 9.87\sigma_y + 0.573\sigma_x^2 + 1.32\sigma_y^2 + 6.25\tau^2 - 0.30\sigma_x\sigma_y \\ &\quad + 0.009585\sigma_x^2\sigma_y + 0.003135\sigma_x\sigma_y^2 + 0.28398\sigma_x\tau^2 \\ &\quad + 0.4689\sigma_y\tau^2 = 1 \end{aligned} \tag{12}$$

The validity of the yield criterion is demonstrated by comparing the derived analytical yield surface of Eq. (12) with the existing experimental results of Page (1981). More than 70 points of experimental data have been depicted in Fig. 2. In the same figure, analytical curves in principal stress terms are also depicted for the yield surface of Eq. (12). The good agreement between the analytical and experimental data is apparent for this general yield surface with a nonsymmetric curve.

Computer Code

In order to implement the proposed method of analysis, a specific finite-element computer program for the 2D nonlinear analysis of a masonry plane wall, under monotonic static loading, has been developed. In developing the code, use of the ready-made databanks of the Owen and Hinton PLAST computer code (Owen and Hinton 1982) has been made.

Many researchers have used the Owen and Hinton's code in order to develop new software for the nonlinear analysis of masonry. The most representative of these is a nonlinear analysis computer code developed by Andreaus (1996). The main disadvantages of the Owen and Hinton software are the isotropic consideration of the materials and the use of isotropic yield criteria.

The software developed in the present work overcomes the aforementioned disadvantages of PLAST and is appropriate to model the anisotropic behavior of masonry, allowing use of the regular yield surface developed [Eq. (12)].

In the development phase, special attention has been given in producing a visual representation of the analysis procedure and of the response results. In particular, the user is able to follow the individual stages of the analysis as it progresses (i.e., for each load increment), by observing:

- The flow-chart diagram produced on the screen, which gives information on the individual operation performed within the program, the number of iterations needed for convergence within each load increment, and the computer run time required;
- The load-displacement diagram; and
- The colored graphic images of the yield process, produced for each individual element within the structure according to the type of stresses combination under which yield takes place [i.e., yield under biaxial compression, tension, or a heterosemous (from the Greek *heteros* = other, different, and *sema* = sign) stresses combination].

This visual representation of the analysis procedure is particularly useful to the user, not only for the instant information it provides as the software runs, but also for the verification of the results produced.

Verification of the Model

To illustrate the effectiveness of the described procedure, the results are compared with available solutions obtained from the literature from experimental tests. Specifically, the capacity of a masonry panel acting as a deep beam (Fig. 3) has been investigated. Page (1978) tested a masonry deep beam with dimensions of 757 × 457 mm, rigidly supported at each end over a 188 mm length. The panel was made of half scale, pressed solid clay bricks 122 × 37 × 54 mm in dimension, with 5-mm-thick mortar joints (1:1:6 cement:lime:sand by volume).

Vertical stress distributions at level A-A (see Fig. 3), obtained from experiments performed by Page (1978), are compared to the results obtained using our proposed methodology. Stress distributions at one level of applied load, *P*, are shown in Fig. 4. Results obtained from a conventional finite-element analysis, with the masonry modeled as a continuum and with average properties and isotropic elastic behavior, are also included.

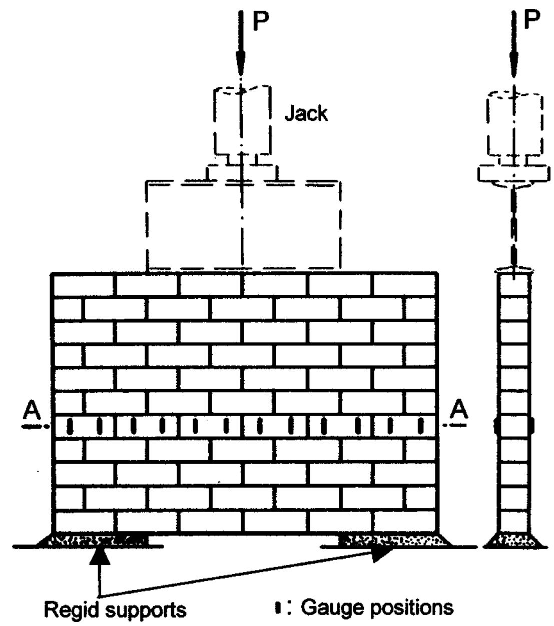

Fig. 3. General arrangement of masonry deep beam test (Page 1978)

Clearly, there is a good agreement between the results of the present inelastic analysis and the experimental results. The elastic solution deviates grossly from both the experimental and inelastic results, and this confirms that the material behavior is significantly nonlinear.

Graphic images of the yield process can be produced for every increment of load (Fig. 5). These images are colored according to

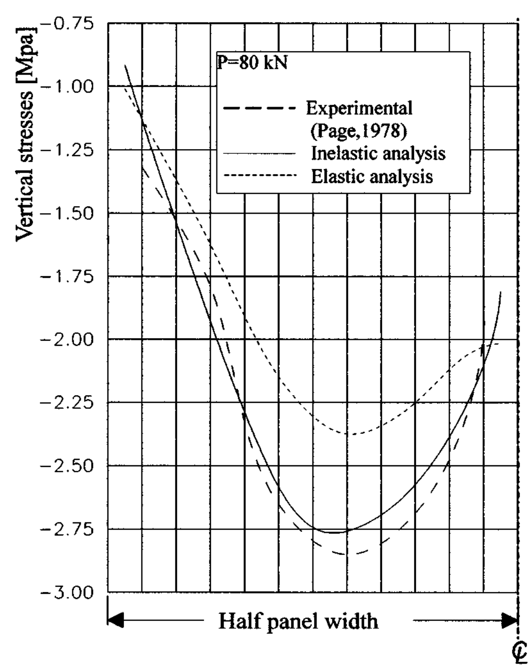

Fig. 4. Vertical stresses distributions in masonry deep beam

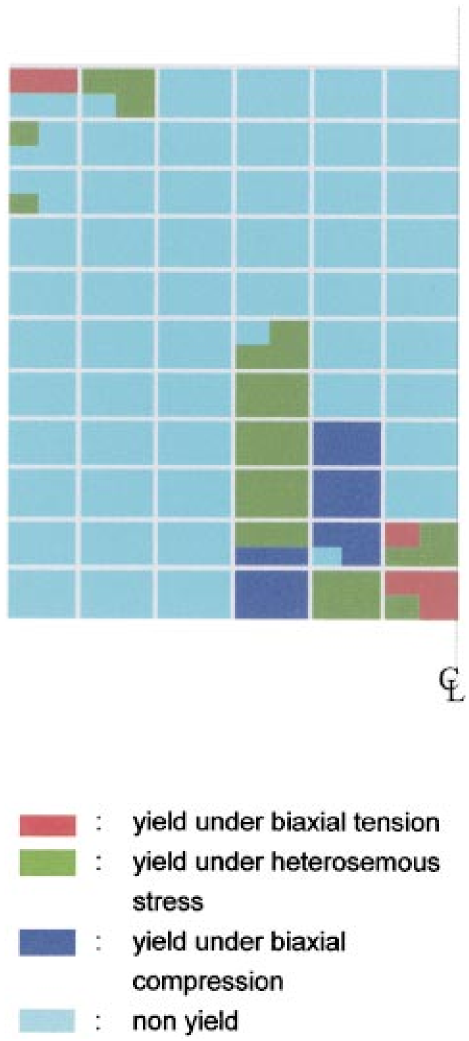

Fig. 5. (Color) Yield pattern for masonry deep beam

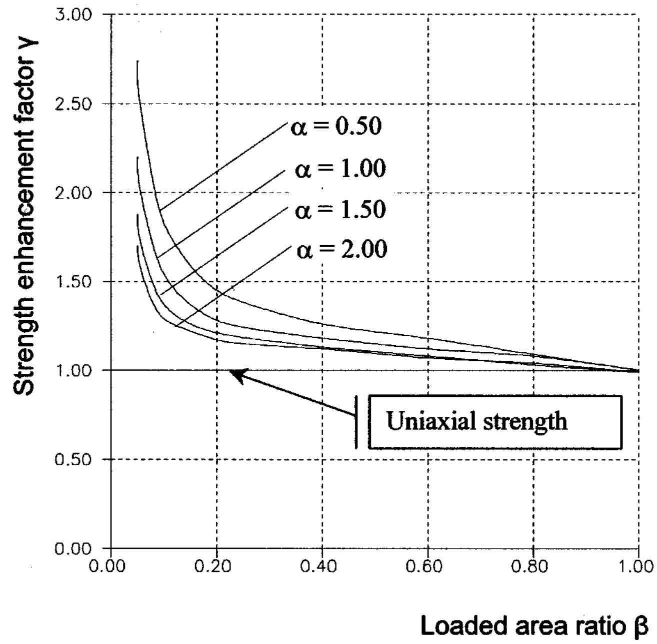

Fig. 6. Variation of strength enhancement factor γ as a function of loaded area ratio β for various values of aspect ratio α

the type of stress under which yield takes place for every particular element (i.e., yield under biaxial compressive, tensile, or a combination of heterosemous stresses). These graphic images are especially useful not only because of the visual information they offer, but also because of the validation they provide. For example, the deviation of the inelastic analysis curve (which is in close agreement to the experimental results) relative to the elastic analysis curve, is in good agreement with the yield pattern (Fig. 5) in which yield occurs in the area of the highest elastic-inelastic curve deviation.

Parametric Investigation

A total of 62 unreinforced masonry walls subjected to concentrated compression load were analyzed using the developed computer code. A distributed load on a very small area a' ($a'/a = 0.05$ or 0.10) has been considered. The influence of the loaded area ratio β, the load location d/a, and the wall aspect ratio were investigated with the following assumptions:

- The wall is perfectly fixed at the ground level and is acted upon by a concentrated compression load, which is proportionally increased up to failure, according to the load factor λ. The reference load amplitude is assumed equal to 0.1 MPa, and the load factor increment is assumed equal to 0.5.
- The masonry wall has been discertized by means of four-node isoparametric quadrilateral elements, with a finer mesh immediately below the loaded area, where stress gradients are high. A typical finite-element mesh is shown in Fig. 1(b).
- Anisotropic behavior has been assumed for the masonry material, with moduli of elasticity $E_x = 4{,}500\ \mathrm{N/mm^2}$ and $E_y = 7{,}500\ \mathrm{N/mm^2}$ and Poisson's ratios $\nu_{xy} = 0.19$ and $\nu_{yx} = 0.32$.

Central Concentrated Load

A total of 32 centrally loaded masonry walls were analyzed, with aspect ratios ranging from 0.50 to 2.00 (increasing with a 0.50 step) and loaded area ratios ranging from 0.05 to 1.00.

Fig. 6 depicts the variation of the strength enhancement factor γ as a function of the factor β, for various values of the parameter α. For low values of the β factor ($\beta < 0.30$), the increment of the strength enhancement factor is excessively high. These values can be much higher in the case of low values of the factor α($\alpha = 0.50$).

The influence of the wall aspect ratio α on the strength enhancement factor γ for various values of the loaded area ratio β is presented in Fig. 7. The reduction of the wall aspect ratio α value leads to an approximately linear increment of the wall strength. More significant is the increment for low values of the β factor. For $\beta = 0.05$, the wall's strength is reduced up to 60% when the factor α increases from 0.50 to 2.00.

Eccentric Concentrated Load

A total of 30 loaded masonry walls were analyzed, with aspect ratios of 0.50, 1.00, and 1.50. The loaded area ratio has been investigated for the values 0.05 and 0.10 (characteristic values for concentrated loads). Values of load eccentricity ratio d/a from 0.00 to 0.50 have been taken into consideration.

In Fig. 8, the variation of the strength enhancement factor γ as a function of the concentrated load eccentricity, d/a, for the case

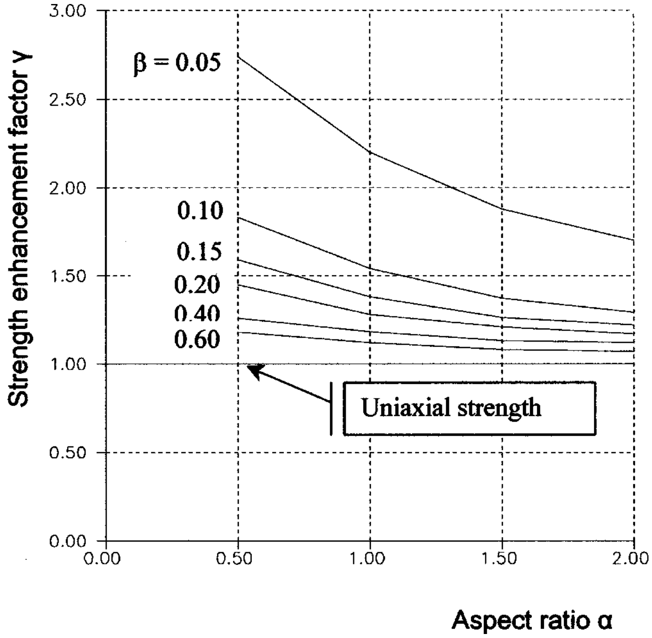

Fig. 7. Variation of strength enhancement factor γ as a function of aspect ratio α for various values of loaded area ratio β

$\alpha = 1.00$ is presented (similar results are obtained for the case of $\alpha = 0.50$). According to these figures, the increment of the load eccentricity (reduction of ratio d/a) leads to a reduction of the wall strength. For load eccentricity values between 0.20 and 0.50 the strength reduction is almost zero. This reduction becomes more significant for values of the ratio d/a between 0.00 and 0.20. The strength enhancement factor γ can get values lower than 1 (Fig. 8), in accordance with existing experimental results (Arora 1988).

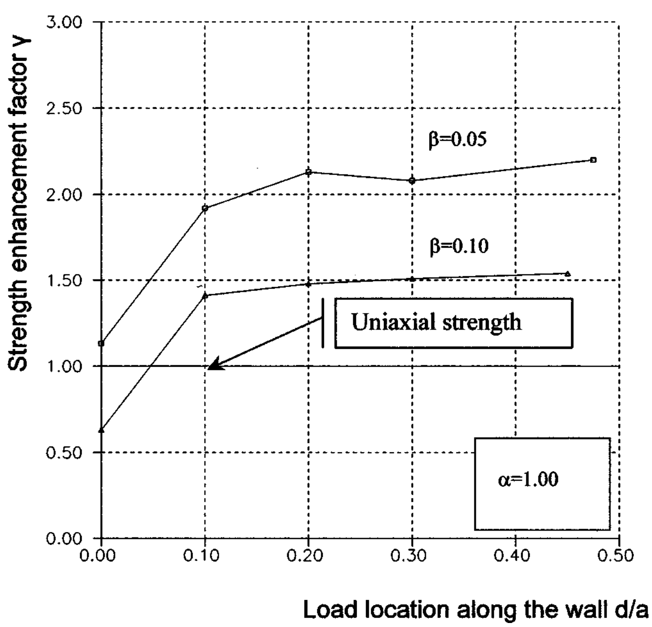

Fig. 8. Strength enhancement factor of eccentric loading

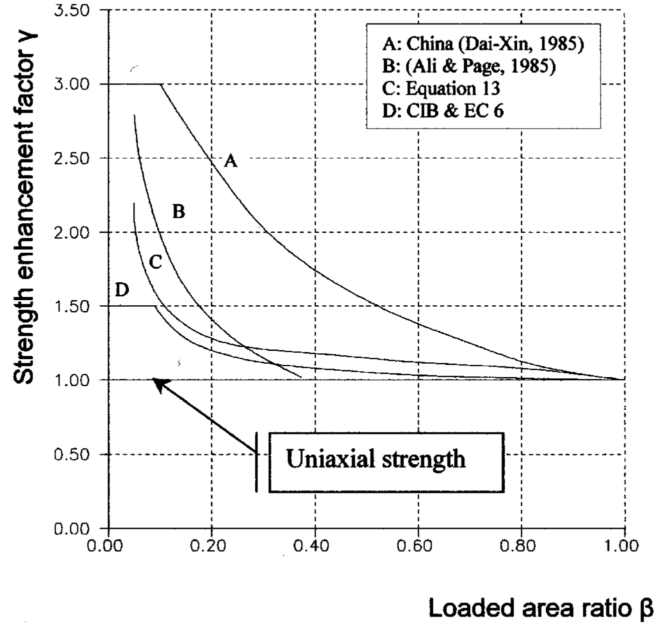

Fig. 9. Typical design provisions for concentrated loads ($\alpha = 1.00$; central load)

Proposed Design Recommendation

A general design rule for the strength enhancement factor should allow for the influence of the loaded area ratio β, the load location along the wall d/a, and the aspect ratio of the wall α. Using the results of the present parametric study, the following relationship for the strength enhancement factor γ can be suggested:

$$\gamma = \varphi\left(1.00 + \frac{2}{\alpha}\frac{(1-\beta)}{(1+3\beta)^4}\right) \qquad (13)$$

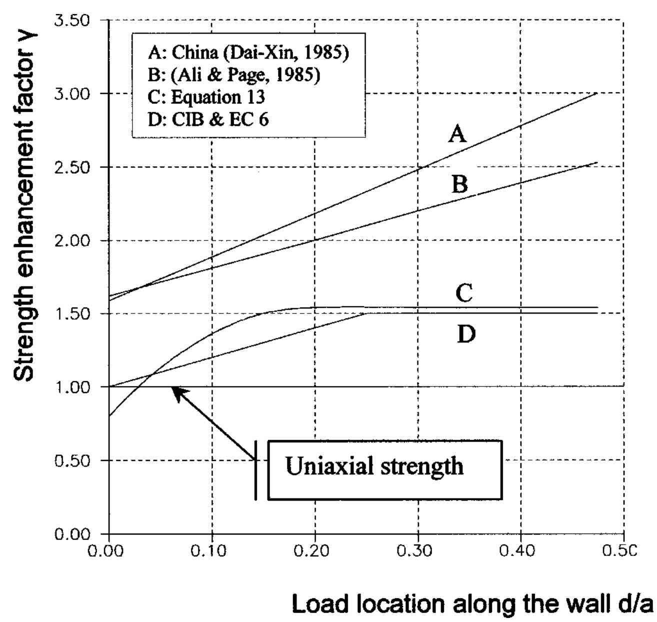

Fig. 10. Typical design provisions for concentrated loads ($\alpha = 2.00$; $\beta = 0.05$; eccentric load)

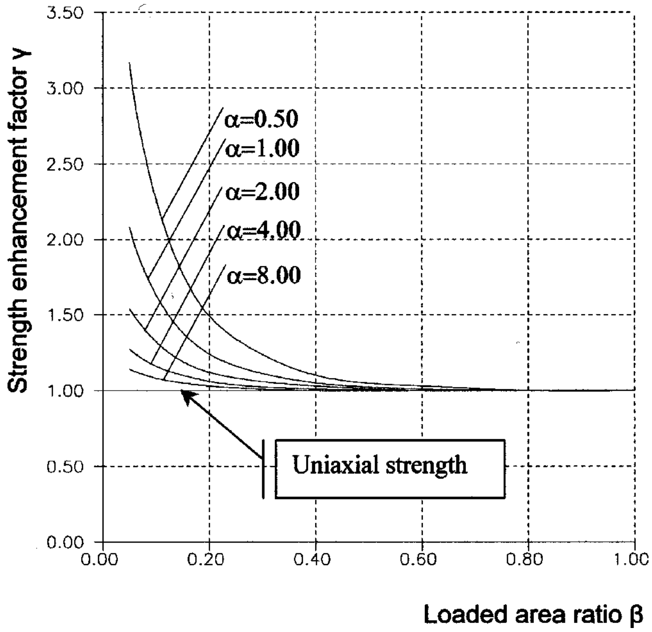

Fig. 11. Proposed design recommendation for concentric load

where

$$\varphi = 1.00 \quad \text{for the case of } 0.20 \leqslant \frac{d}{a} \leqslant 0.50$$

$$\varphi = 1.00 - 12\left(0.20 - \frac{d}{a}\right)^2 \quad \text{for the case of } 0.00 \leqslant \frac{d}{a} \leqslant 0.20$$

In Figs. 9 and 10, the graphical representation of Eq. (13) (curve C) is compared to the variation of the γ-values provided by Dai-Xin (curve A), Ali and Page (curve B), and EC6 (curve D). Fig. 9 demonstrates the comparison of the results of the different analytical models; equations proposed by Chinese code GBJ-3-73 (curve A) as well as by Ali and Page (curve B) overestimate the strength of the wall, as compared to the Eurocode equation (curve D) and our proposed equation (curve C). All the aforementioned equations limit themselves to yield values greater than 1 (>1) of the strength enhancement factor γ. The results overcome this limitation and confirm the experimentally obtained values of $\gamma<1$, for the eccentric load shown experimentally by Arora (1988).

Fig. 11 [which is also derived from Eq. (13)] shows the variation of the strength enhancement factor γ with the loaded area ratio β, for various values of the wall aspect ratio α (8.00,4.00,2.00,1.00,0.50) and central loading.

It is to be noted that, as the presented results are given in nondimensional terms, expressions similar to Eq. (13) can be derived for other materials as well, e.g., concrete, by following the same process, with different input data (e.g., mechanical characteristics).

Conclusions

An analytical parametric study of the behavior of masonry walls subjected to concentrated loads has been presented. An iterative (incremental) finite-element model has been used to simulate the wall behavior. The nonlinear deformation of masonry can be analyzed by means of this model.

In the parametric study, the influence of the loaded area ratio, the position of the load, and the wall geometry on the strength enhancement factor, γ, have been considered. All variables have been shown to influence significantly the degree of enhancement of the bearing strength appearing in the area beneath the load.

Using the results of the parametric study, a new design rule, incorporating all the aforementioned three variables, has been proposed. This new rule, as compared to the existing design recommendations (which do not consider all variables), has proved to be more accurate and closer to the experimental results.

Notation

The following symbols are used in this paper:

a = length of wall;
a' = length of loading plate;
d = distance of concentrated load from nearest corner of wall;
α = aspect ratio of wall;
$\beta(=a'/a)$ = loaded area ratio (ratio of loaded area to total area);
γ = strength enhancement factor; and
λ = load factor.

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