Non-Linear Analysis of Masonry Shear Walls

by

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ABSTRACT

A new methodology for the non-linear macroscopic analysis of unreinforced masonry (URM) shear walls under biaxial stress state is presented, using the finite element method. The methodology focuses on the definition / specification of the yield surface for the case of anisotropic masonry under biaxial stress state, as well as on the numerical solution of this non-linear problem. Specifically, in order to define the yield surface, we use a cubic tensor polynomial, and apply the initial stress method in order to solve the elasto-plastic problem. In addition, a novel computer code of finite elements has been developed, in order to implement the proposed method of analysis. The main advantage of the method is that the formulation of the plasticity equations through a regular yield surface, leads to the elimination of the problem that occurs by the use of a singular surface. Furthermore, it is clearly shown that the non-linear behaviour of URM is strongly affected by the yield criterion used.

NOTATION

D	=	Elasticity matrix
D_{ep}	=	Elasto-plastic matrix
E	=	Young's modulus for isotropic assumption of the material behaviour
E_x , E_y	=	Module of elasticity in the x and y direction respectively
F_i	=	Strength tensor second rank
F_{ij}	=	Strength tensor fourth rank
F_{ijk}	=	Strength tensor sixth rank
G_{xy}	=	Shear modulus in the xy plane
α	=	Flow vector
λ	=	Plastic multiplier as well as load factor
ν	=	Poisson's ratio for isotropic assumption of the material behaviour
ν_{xy} , ν_{yx}	=	Poisson's ratios in the xy and yx plane respectively
σ_x, σ_y	=	Normal plane stress along x-axis and y-axis respectively
τ	=	Shear stress measured in the x, y plane

1. INTRODUCTION

Although URM walls exhibit distinct directional properties, due to the influence of mortar joints acting as planes of weakness, it is only recently that analytical procedures which account for the non-linear anisotropic behaviour of masonry under static loads, have been developed. These analytical procedures could be summarized in the following two levels of refinement for masonry models:

- ***Macro-modelling (masonry as an one-phase material)***. According to this procedure [1, 2], no distinction between the individual units and joints is made, and masonry is considered as a homogeneous, anisotropic continuum. While this procedure may be preferred for the analysis of large masonry structures, it is not suitable for the detailed stress analysis of a small panel, due to the fact that it is difficult to capture all its failure mechanisms. The influence of the mortar joints acting as planes of weakness cannot be addressed.

- ***Micro-modelling (masonry as a multy-phase material)***. According to this procedure [3-6], the units, the mortar, and the unit/mortar interface, are modeled separately. While this leads to more accurate results, the level of refinement means that any analysis will be computationally intensive, and so will limit its application to small laboratory specimens and structural details. A simplified micro-modelling procedure has been recently proposed by Sutcliffe et al. [7]. According to this procedure, which is an intermediate approach, the properties of the mortar and the unit/mortar interface (masonry as a two-phase material) are lumped into a common element, while expanded elements are used to represent the brick units. Although this approach leads to less accurate results, the reduction in computational intensiveness, yields a model, which would be applicable to a wider range of structures.

In the present work, a new methodology for the non-linear analysis of anisotropic masonry shear walls under biaxial stress state is presented, regarding masonry as an one-phase material. The basic assumptions and the associated mathematical expressions of the theory of plasticity are outlined, giving special attention to the case of masonry structures. More specifically, in order to formulate the quantitative expressions of the mathematical theory of plasticity, a new analytic method has been used for the description of the yield of the anisotropic masonry via a **regular** surface, that is, a surface defined by a single equation of the form $f(\sigma)=0$ [8].

The significance of the use of a regular yield surface has been manifested since 1950, and introduced by Hill in his book "The Mathematical Theory of Plasticity" [9]. The theory of plasticity through a closed yield surface encounters the existence of singular points on the yield surface. According to Zienkiewcz et al. [10], this problem imposes additional computational difficulties in the non-linear analysis procedure.

Another problem of the up-to-date non-linear analysis of the behaviour of masonry, is the use of ready-made analysis software packages that have been developed mainly for the case of concrete [11, 12]. The main disadvantage of these ready-made software packages is that their architecture is not amenable to modifications and, therefore, they cannot take into account important features, which are appropriate for the case of masonry.

To overcome these problems, a novel computer code, in FORTRAN programming language, has been developed for the structural design of URM shear walls. The code can be applied for the analysis of an elasto-plastic anisotropic URM wall, under plane stress. During the development procedure, special attention has been given at the graphic imaging of the analysis results. The software possesses the capability of automatic mesh generation, and produces the load – displacement diagram, giving a coloured image of the yield pattern within the structure, for every increment of load.

2. BASIC MATHEMATICAL ASPECTS OF THE NON-LINEAR ANALYSIS

In order to formulate a theoretical description, capable to model elasto-plastic material deformation, three requirements have to be met:

- An explicit relationship between stress and strain that will describe the material's behaviour under elastic conditions must be expressed,
- A yield criterion that will define the stress level at which plastic flow commences must be postulated, and
- A relationship between stress and strain must be developed for post-yield behaviour; i.e., when the deformation is made up of both elastic and plastic components.

The relationship between stress and strain, before the onset of plastic yielding, is given by the following standard linear elastic expression:

$$\sigma = D\varepsilon \tag{1}$$

In this expression σ and ε are the stress and strain components, respectively, and D is the elasticity matrix.

In order to take into account the distinct anisotropic/orthotropic behaviour of masonry, that exhibits directional properties due to the influence of mortar joints acting as planes of weakness, the material is assumed to be homogeneous and anisotropic. In particular, the material of masonry shows a different modulus of elasticity (E_x) in the x direction (direction parallel to the bed joints of masonry) and a different modulus of elasticity (E_y) in the y direction (perpendicular to the bed joints). In the case of plane stress, the elasticity matrix is defined by

$$D = \begin{bmatrix} \frac{E_x}{1-\nu_{xy}\nu_{yx}} & \frac{E_x\nu_{yx}}{1-\nu_{xy}\nu_{yx}} & 0 \\ \frac{E_y\nu_{xy}}{1-\nu_{xy}\nu_{yx}} & \frac{E_y}{1-\nu_{xy}\nu_{yx}} & 0 \\ 0 & 0 & G_{xy} \end{bmatrix} \tag{2}$$

in which ν_{xy}, ν_{yx} are the Poisson's ratios in the xy and yx plane respectively; and G_{xy} is the shear modulus in the xy plane. It is worth noticing that in the case of plane stress in an anisotropic material the following equation holds

$$E_x\nu_{yx} = E_y\nu_{xy} \tag{3}$$

2.1. The yield criterion

The yield criterion defines the stress level at which plastic deformation begins and takes the form of the equation:

$$f(\sigma) = 0 \tag{4}$$

where f is a function.

The geometry of the yield surface tends to have a significant influence not only in the formulation, but also in the numerical solution of the non-linear problem, as we will show in the next paragraph, where we will present in detail all the aspects relevant to the estimation of the elasto-plastic matrix.

2.2. Plastic flow Rule

Von Mises first suggested, in 1928, the basic constitutive relation that defines the plastic strain increments in relation to the yield surface. Various other researchers [13,14] have proposed heuristic methods for the validation of Von Mises relationship. These methods have led to the current state-of-the-art hypothesis, which states that:

If $\delta\{\varepsilon\}_p$ denotes the increment of plastic strain, then:

$$\delta\{\varepsilon\}_p = \lambda \frac{\partial f}{\partial\{\sigma\}} \tag{5}$$

where λ is a determinable constant (plastic multiplier).

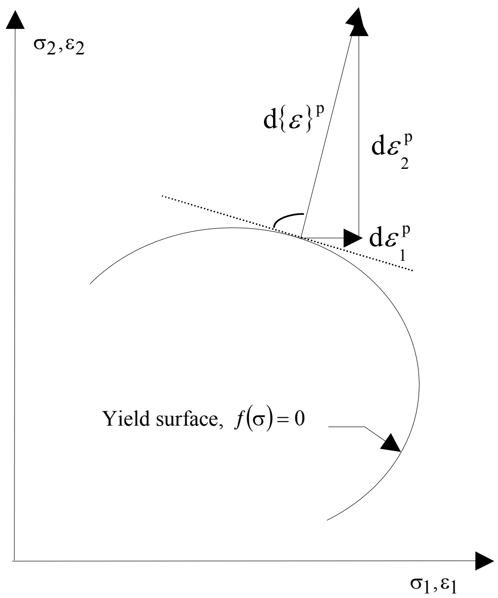

Figure 1-Geometrical representation of the normality rule in 2D Stress Space

This rule is widely known as the normality principle because the relation (5) can be interpreted as requiring the normality of the plastic strain increment vector to the yield surface in the hyper-space of ν stress dimensions. In Figure 1, this normality rule is shown, in the case of a two dimensional space.

2.3. Stress–strain relations

During an infinitesimal increment of stress, changes of strain are assumed to be partly elastic and partly plastic as

$$\delta\{\varepsilon\} = \delta\{\varepsilon\}_e + \delta\{\varepsilon\}_p \tag{6}$$

The elastic strain increments are related to the stress increments via a symmetric matrix of constants [D] known as the elasticity matrix:

$$\delta\{\varepsilon\}_e = [D]^{-1}\delta\{\sigma\} \tag{7}$$

Expression (6) can be readily rewritten as

$$\delta\{\varepsilon\} = [D]^{-1}\delta\{\sigma\} + \frac{\partial f}{\partial\{\sigma\}}\lambda \tag{8}$$

Manipulation of the above equations leads to the following elasto-plastic stress-strain relation:

$$\delta\{\sigma\} = D_{ep}\delta\{\varepsilon\} \tag{9}$$

where

$$D_{ep} = D - \frac{D\alpha\alpha^T D}{\alpha^T D\alpha} \tag{10}$$

is the elasto-plastic matrix, and

$$\alpha = \left\{ \frac{\partial f}{\partial \{\sigma\}} \right\} \tag{11}$$

is the flow vector.

3. YIELD SURFACE GEOMETRY EFFECT IN NON-LINEAR SOLUTION

As we have already mentioned, the yield criterion affects the formulation of the non-linear problem. In this paragraph we will describe the effect the yield surface geometry has in the formulation and the numerical solution of the elasto-plastic problem.

3.1. "Corners" in a yield surface

Sometimes the yield surface is not defined by only a single continuous (and convex) function, but by a series of functions:

$$f_1, f_2, \ldots f_n$$

According to Koiter [8], a surface of this kind is called singular. Such a surface is the yield surface of Tresca and the yield surface for masonry, in three mutually intersected cones, proposed by Dhanasekar, Page, and Kleeman [15].

For most of the bounding surface, only a single condition such as $f_m = 0$ can define the yield surface, and the previously written flow rules apply.

At a singular point ("corner") of the yield surface we may have, however, the condition that

$$f_h = \ldots = f_m = 0$$

For such a singular point, the use of the following equation has been proposed by Koiter [8], for the estimation of the increment of plastic strain, instead of equation 5:

$$d\{\varepsilon_p\} = \lambda_h \left\{ \frac{\partial f_h}{\partial \{\sigma_h\}} \right\} + \ldots + \lambda_m \left\{ \frac{\partial f_m}{\partial \{\sigma_m\}} \right\} \tag{12}$$

where λ_i are positive constants (Figure 2).

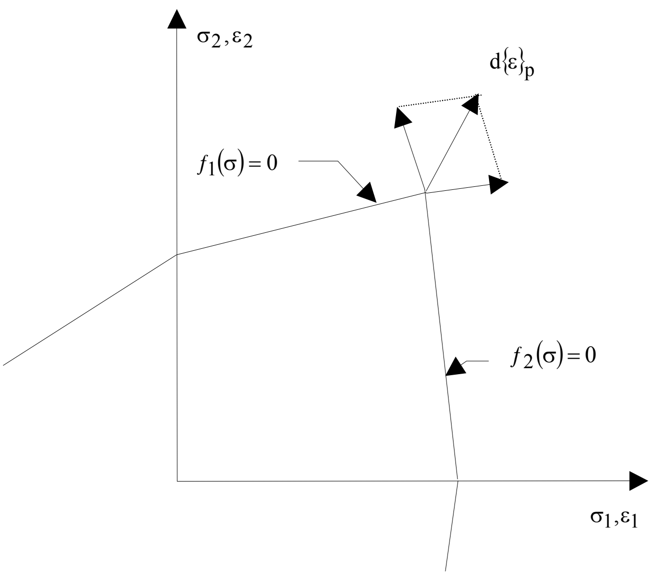

Figure 2-Corners in a yield surface. Graphical interpretation of Koiter's criterion

According to Zienkiewicz, Valliapan, and King [10], the use of singular areas imposes important problems to the elasto-plastic analysis process. The authors propose to avoid calculating the singular points in a yield surface, by making a suitable choice of continuous surfaces, which can usually represent the true condition with a good degree of accuracy.

3.2. Regular yield surface

Having in mind the computational problems introduced during the formulation and the numerical solution of the non linear problem by the use of singular yield surfaces, we will make use of a recently new proposed method by Syrmakezis and Asteris [16] in order to define the yielding for the case of anisotropic masonry. According to this method, a **regular** yield surface is used; that is, a surface defined by a single equation of the form:

$$\begin{aligned} f(\sigma_x, \sigma_y, \tau) = {} & F_1\sigma_x + F_2\sigma_y + F_{11}\sigma_x^2 + F_{22}\sigma_y^2 + \\ & F_{66}\tau^2 + 2F_{12}\sigma_x\sigma_y + 3F_{112}\sigma_x^2\sigma_y + \\ & 3F_{122}\sigma_x\sigma_y^2 + 3F_{166}\sigma_x\tau^2 + 3F_{266}\sigma_y\tau^2 - 1 = 0 \end{aligned} \tag{13}$$

Eliminating all third order terms in Eq. 13, a simplified yield criterion can be derived:

$$\begin{aligned} f(\sigma_x, \sigma_y, \tau) = {} & F_1\sigma_x + F_2\sigma_y + F_{11}\sigma_x^2 + F_{22}\sigma_y^2 + \\ & F_{66}\tau^2 + 2F_{12}\sigma_x\sigma_y - 1 = 0 \end{aligned} \tag{14}$$

This latter simple form of the yield criterion, has already been used by other investigators [1, 15], to define the failure of brick masonry under biaxial stress state.

According to Syrmakezis and Asteris [16], the general yield criterion (Eq. 13) through its non-symmetric form, fit the non-symmetrically dispersed experimental data better than the simplified model (Eq. 14).

Using the above yield surfaces, the expression of flow vector is defined by

$$\alpha = \begin{Bmatrix} \dfrac{\partial f}{\partial \sigma_x} \\ \dfrac{\partial f}{\partial \sigma_y} \\ \dfrac{\partial f}{\partial \tau} \end{Bmatrix} = \begin{bmatrix} F_1 + 2F_{11}\sigma_x + 2F_{12}\sigma_y + 6F_{112}\sigma_x\sigma_y + \\ +3F_{122}\sigma_y^2 + 3F_{166}\tau^2 \\ F_2 + 2F_{22}\sigma_y + 2F_{12}\sigma_x + 3F_{112}\sigma_X^2 + \\ +6F_{122}\sigma_x\sigma_y + 3F_{266}\tau^2 \\ 2F_{66}\tau + 6F_{166}\sigma_x\tau + 6F_{266}\sigma_y\tau \end{bmatrix} \tag{15}$$

for the case of the general yield criterion, and by

$$\alpha = \begin{Bmatrix} \dfrac{\partial f}{\partial \sigma_x} \\ \dfrac{\partial f}{\partial \sigma_y} \\ \dfrac{\partial f}{\partial \tau} \end{Bmatrix} = \begin{bmatrix} F_1 + 2F_{11}\sigma_x + 2F_{12}\sigma_y \\ F_2 + 2F_{22}\sigma_y + 2F_{12}\sigma_x \\ 2F_{66}\tau \end{bmatrix} \tag{16}$$

for the case of the simplified yield criterion.

4. COMPUTER CODE

In order to implement the method, a specific computer program for 2D non-linear finite element analysis of a masonry plane wall under monotonic static loading, has been developed. During the development procedure, we have made use of the ready-made databanks of Owen & Hinton PLAST computer code [17].

It must be mentioned that many other researchers used Owen & Hinton's code, in order to develop new software for the non-linear analysis of masonry. The most representative of these, is a non-linear analysis computer code developed by Adreaus [1]. The main disadvantages of the Owen & Hinton software, is the isotropic consideration of the materials and the use of isotropic yield criteria.

For the non-linear solution of the problem, the method of initial stiffness proposed by Zienkiewicz, Valliapan and King [10], has been used. According to this method, the solution of an elasto-plastic problem is based on a series of successive approximations.

The software used in the present work, overcome the above-mentioned disadvantages of PLAST, and is appropriate to model the anisotropic behaviour of masonry, allowing the use of the regular yield surfaces developed (Eqs. 13 and 14). During the development phase, special attention has been given to the automatic graphic representation of the analysis results.

5. NUMERICAL EXAMPLE

Using the computer program developed, we studied the non-linear behaviour of a URM shear wall with openings, as shown in Figure 3, under uniform compressive and shear loading, and with the following assumptions:

- The wall is perfectly fixed at the ground level and is acted upon by both horizontal and vertical loads, which are proportionally increased up to failure, according to the load factor λ. The loads are uniformly distributed at the wall top; the reference load amplitude in both directions is assumed to equal 0.1 N/mm^2 and the load factor increment, equal to 0.1.
- The masonry wall has been discretized by means of four-node isoparametric quadrilateral elements, whose length is 1.00 m.
- Both isotropic and anisotropic behaviour has been assumed for the masonry material, with Young's modulus E=5700 N/mm2 and Poisson's ratio ν=0.19, for the isotropic case study, and moduli of elasticity E_x=4500 N/mm^2 and E_y=7500 N/mm^2 and Poisson's ratios ν_{xy}=0.19 and ν_{yx}=0.32 for the anisotropic case study.
- Both the simplified and the general yield criterion developed, has been used.

For the mechanical characteristics of the masonry, we have used the experimental results of Page [18]. Using these results, the regular yield surface, for the case of the general yield criterion (Eq. 13), can be defined as [2, 16]:

$$2.27\sigma_x + 9.87\sigma_y + 0.573\sigma_x^2 + 1.32\sigma_y^2 + 6.25\tau^2 - 0.30\sigma_x\sigma_y + 0.009585\sigma_x^2\sigma_y + 0.003135\sigma_x\sigma_y^2 + 0.28398\sigma_x\tau^2 + 0.4689\sigma_y\tau^2 = 1 \quad (17)$$

In addition, for the simplified yield criterion (Eq. 14), the regular yield surface can be defined as:

$$2.27\sigma_x + 9.87\sigma_y + 0.573\sigma_x^2 + 1.32\sigma_y^2 + 6.25\tau^2 - 0.454\sigma_x\sigma_y = 1 \quad (18)$$

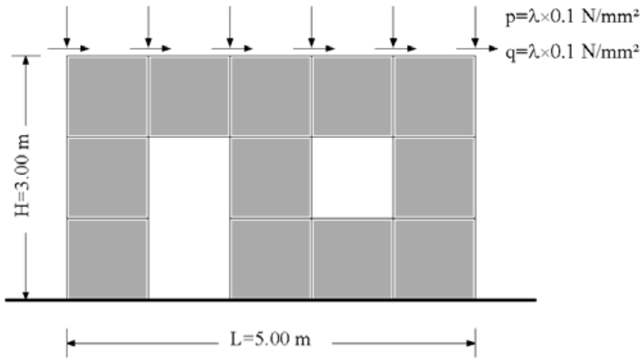

Figure 3-Unreinforced masonry shear wall with openings under both horizontal and vertical loads

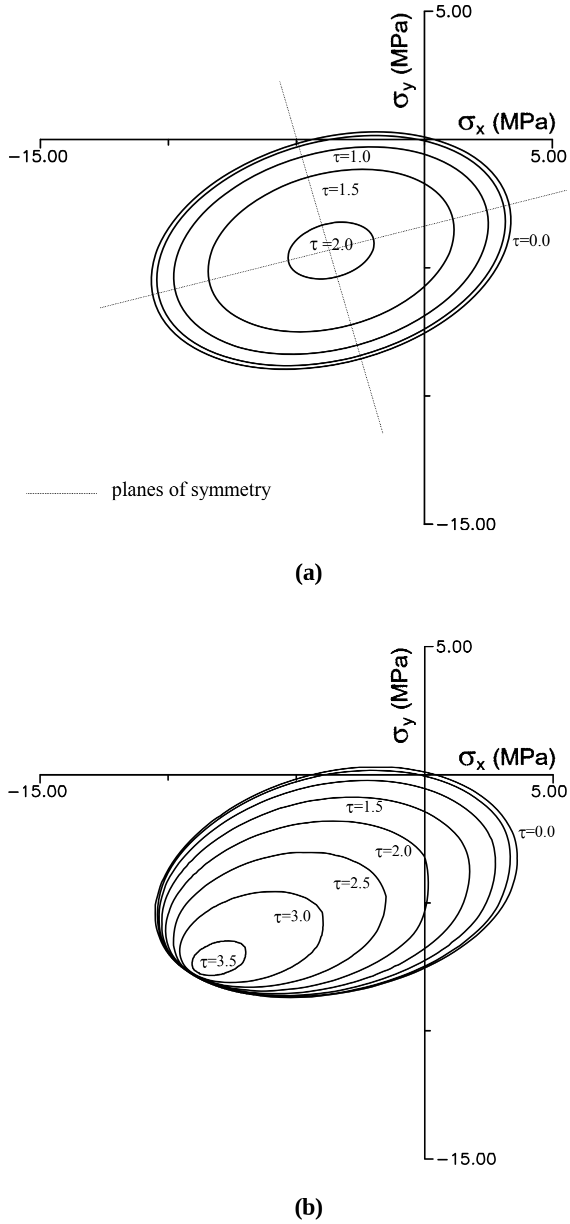

Figure 4-Yield surface of masonry in normal stress terms: (a) Simplified yield criterion; (b) general yield criterion

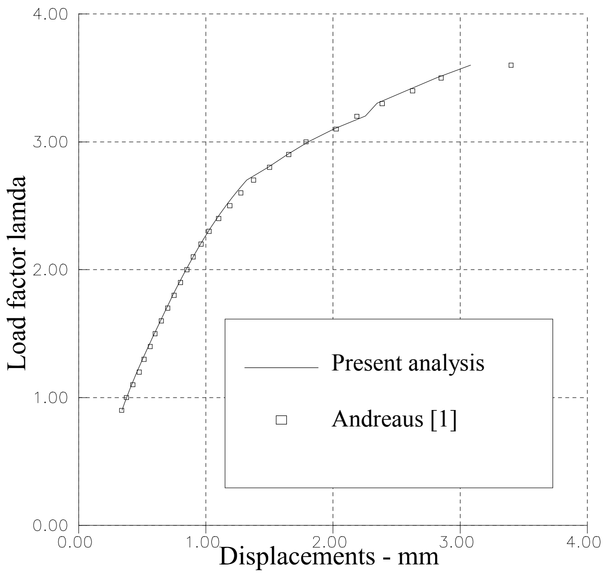

Figure 5-Load factor lamda (λ)-displacement diagram

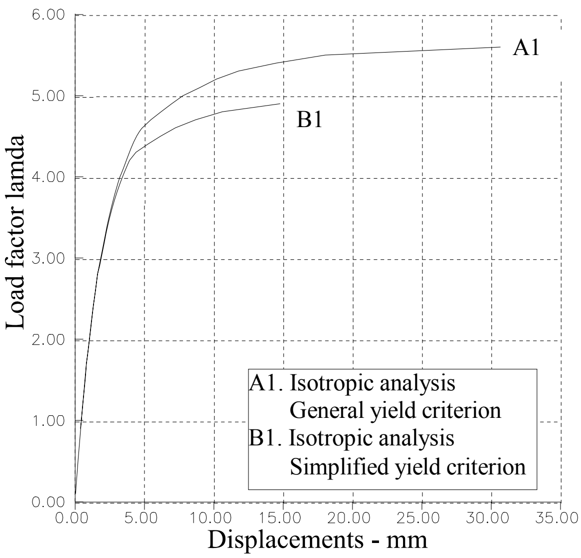

Figure 6-Load factor lamda (λ)-displacement diagram

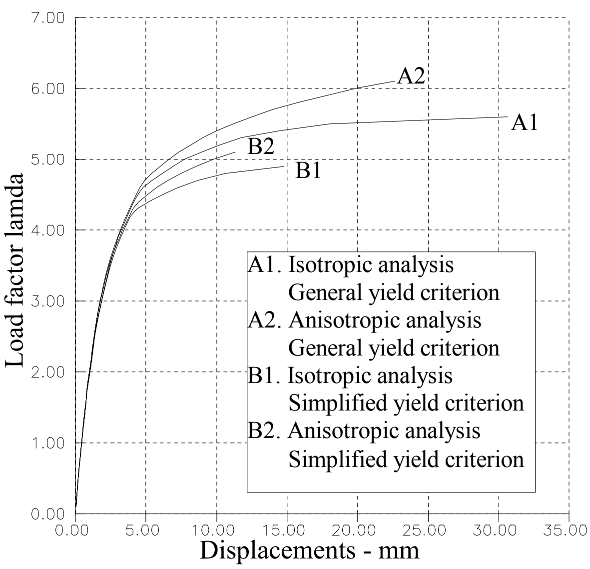

Figure 7-Load factor lamda (λ)-displacement diagram

The regular yield surface described, is depicted in Figure 4a, whereas the simplified yield surface, is depicted in Figure 4b respectively.

Using the method described, the diagram of the load factor λ versus displacements at the top of the masonry wall is computed, and compared, in Figure 5, with the corresponding analytical results taken from Andreaus [1]. It is clearly shown that there is a good agreement between the results of the present non-linear analysis and the analytical results of Andreaus [1]. This was expected due to the fact that the same assumptions have been made for the analyses.

Figure 6 shows the diagram of the load factor λ versus displacement, using both the simplified and the general yield criterion described, and assuming the masonry material to be isotropic. It is clear that the non-linear behaviour of the masonry is affected by the yield criterion. It must be noted that a strong variation in the results appears, although both criteria used, have the same mechanical masonry characteristics (same mono-axial compressive and tensile strength as well as the same strength in pure shear). It is also clear from Figure 7, that that the non-linear behaviour of masonry is affected not only by the yield criterion but also by the anisotropy of the masonry material. In particular, the load factor λ versus displacement diagram is given, using both the simplified and the general yield criterion, and assuming isotropic or anisotropic behaviour for the masonry material. It is clearly shown that the anisotropic behaviour of masonry leads to a greater ultimate load (lines A2 and B2) and to a less ultimate displacement. Thus, the consideration of anisotropy of the masonry material, leads to a more brittle behaviour of masonry.

6. CONCLUSIONS

The present work applies a new methodology for the non-linear 2D finite element analysis of anisotropic masonry under monotonic loading. The methodology focuses on the definition / specification of the yield surface for the case of anisotropic masonry under biaxial stress state, as well as on the numerical solution of this non-linear problem. Specifically, in order to define the yield surface, a cubic tensor polynomial has been used, and the initial stress method has been applied in order to solve the elasto-plastic problem. In addition, a novel computer code of finite elements has been developed, in order to implement the proposed method of analysis, which takes account of the specifically intense anisotropic behaviour of masonry. The main advantage of the method is that the formulation of the plasticity equations through a regular yield surface, leads to the elimination of the problem that occurs by the use of a singular surface. Furthermore, it is clearly shown that the non-linear behaviour of masonry is strongly affected by the yield criterion used, as well as by the anisotropy of its material.

REFERENCES

1. ANDREAUS, U., Failure criteria for masonry panels under in-plane loading, ASCE J. Struct. Engrg., ASCE, **122**, (1), 37-46, 1996.

2. ASTERIS, P. G., *Analysis of anisotropic nonlinear masonry*, PhD thesis, Dept. of Civ. Engrg., National Technical University of Athens, Greece, 2000.

3. PAGE A. W., Finite element model for masonry, J. Str. Div., ASCE, **104**, (8), 1267-85, 1978.

4. ALI, S. and PAGE, A.W., Finite element model for masonry subjected to concentrated loads, J. Str. Div., ASCE, **114**, (8), 1761-84, 1988.

5. TZAMTZIS, A.D., *Dynamic finite element analysis of complex discontinuous and jointed structural systems using interface elements*, PhD Thesis, Department of Civil Engineering, QMWC, University of London, 1994.

6. LOURENCO, P. B., *Computational strategies for masonry structures*, Dissertation, Dept. of Civ. Engrg., Delft University of Technology, 1996.

7. SUTCLIFFE, D. J., YU, H. S. and PAGE, A. W., Lower bound limit analysis of unreinforced masonry shear walls, Computer and Structures, **79**, 1295-1312, 2001.

8. KOITER, W. T., Stress-strain relations, uniqueness and variational theorems for elastic-plastic materials with a singular yield surface, Quarterly of applied mathematics, Vol. **XI**, 350-354, 1953.

9. HILL, R., *The Mathematical Theory of Plasticity*, Oxford University Press, 1960.

10. ZIENKIEWICZ, O. C., VALLIAPANT, S. and KING, I. P., Elasto-platic solutions of engineering problems; ‘initial stress’ finite element approach, Int. J. for Numerical Methods in Engrg., **1**, 75-100, 1969.

11. ANDREAUS, U. and IPPOLITY, L., A two-storey masonry wall under monotonic loading: a comparison between experimental and numerical results, Proceedings of the 4th International conference on structural studies of historical buildings, 319-326, Crete, Greece, 1995.

12. BALLIO, G., CALVI, M. and MAGENES, G., *Experimental and numerical investigation on a brick masonry building prototype*, Reports 1.1, 2.0, 3.0, Consiglio Nazionale delle Ricerche, Gruppo Nazionale per la Difesa dai Terremoti, Italy, 1992.

13. DRUCKER, D. C., A more fundamental approach to plastic stress-strain solutions, Proceedings of the 1st U. S. natn. Cong. Appl. Mech., 487-491, 1951.

14. PRAGER, W., A new method of analyzing stress and strain in work hardening plastic solids, Revue Méc. Appl., Buc., **23**, 493-496, 1956.

15. DHANASEKAR, M., PAGE, A. W. and KLEEMAN, P. W., The failure of brick masonry under biaxial stresses, Proc. Instn Civ. Engnrs, London, England, **79**, (2), 295-313, 1985.

16. SYRMAKEZIS, C. A. and ASTERIS, P. G., Masonry failure criterion under biaxial stress state, ASCE J. Mat. in Civ. Engrg., **13**(1), 58-64, 2001.

17. OWEN, D. R. J., and HINTON, E., *Finite Elements in Plasticity: Theory and Practice*, Pineridge Press Ltd., Swansea, U.K, 1982.

18. PAGE A. W., The biaxial compressive strength of brick masonry, Proc. Instn Civ. Engrs, **71**, (2), 893-906, 1981.