NINTH
NORTH AMERICAN MASONRY CONFERENCE

June 1 – 4, 2003
Clemson, South Carolina, USA
Reviewed in accordance with the policies of The Masonry Society

FINITE ELEMENT ANALYSIS OF MASONRY STRUCTURES: PART II – PROPOSED 3-D NONLINEAR MICROSCOPIC MODEL

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Abstract

Following the comprehensive review of previous work conducted in Part I of this study, a 3-D nonlinear microscopic finite element model for static and dynamic analysis of masonry structures is presented. The model considers masonry as a two-phase material, treating bricks and mortar joints separately, thus allowing for non-linear deformation characteristics and progressive local failure of both bricks and mortar joints. The influence of the mortar joints is taken into account by using 'interface' elements to simulate the time-dependent sliding and separation along the interfaces. Stress distributions from an in-plane bending test on a masonry panel were reproduced to a reasonable degree of accuracy. The potential of the proposed three-dimensional model is also demonstrated by using it to predict the non-linear response of a masonry wall panel to an earthquake excitation. Analytical and experimental solutions available in the literature are employed to verify the results obtained from the present finite element model, showing that it is capable of a high degree of accuracy.

Keywords: masonry structures, finite element analysis, literature review, interface element

Introduction

In Civil Engineering, inherently three-dimensional structures with joints or discontinuities subjected to static, dynamic, or seismic loads, present an interesting but complex problem, especially when their material behavior is non-linear. Masonry structures provide a familiar example of this. Masonry walls, designed as vertical load-bearing elements primarily to

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resist in-plane compressive forces, are frequently subjected to considerable lateral loads, arising from one or more of the following: eccentricity of the applied in-plane forces, earth pressure, wind loads, and seismic or blast forces. Use of three-dimensional finite element analysis requires a considerably greater effort in comparison with processes based on two-dimensional idealisations, which provide convenient and economical solutions. However, many engineering problems demand, by virtue of their configuration, loading or behavior, a three-dimensional solution for a realistic prediction of their behavior. Furthermore, a three-dimensional solution can be, and is sometimes used, to verify the accuracy of the corresponding idealised two-dimensional solution.

The review of previous work on the finite element analysis of unreinforced masonry structures, conducted in Part I of this study, reveals that two-dimensional analyses with plane stress formulation have mainly so far been adopted for the finite element analysis of masonry structures. In the absence of a suitable model to represent its behavior, masonry was assumed to be an isotropic elastic continuum; consequently, the influence of the mortar joints acting as planes of weakness, could not be addressed. Indeed, it is only recently that analytical procedures, which account for the non-linear behavior of masonry under static loads, have been developed.

An extension of these procedures to static and dynamic/seismic response analysis of 3-D masonry structures is reported in this part of the study. In particular, a three-dimensional nonlinear microscopic finite element model for the analysis of unreinforced masonry when subjected to static loading and/or dynamic excitations is presented. The model adopted treats bricks and mortar joints separately and allows for non-linear deformation characteristics and progressive local failure of both bricks and mortar joints. The influence of mortar joints has been taken into account by using interface elements to simulate the time-dependent sliding and separation along interfaces. The proposed method of analysis is applicable to both static and dynamic problems in three-dimensions and is suitable for analysis of response, including the effects of material non-linearity and discontinuous contact surface. It is particularly suited to cases in which extensive stress redistribution occurs due to non-linear material behavior and local failure.

The inelastic non-linear behavior of masonry, subjected to an in-plane bending load, has been studied first. A loading of this nature produces high gradients of stress, with accompanying stress redistribution from local joint failure. The proposed finite element model has then been used to predict the non-linear response of an unreinforced masonry wall panel to an in-plane earthquake excitation. Analytical and experimental solutions available in the literature have been employed to verify the results obtained from the present finite element model.

Three-Dimensional FE Model

For the efficient non-linear analysis of masonry, it is necessary to consider relative slip, debonding and cycles of closing and opening of the interfaces. To account for this behavior, a specially developed three-dimensional interface element (shown in Figure 1) has been used to simulate the mortar joints present in a masonry wall assemblage. The purpose of the element is to permit large relative movements to occur between adjacent blocks, and the transfer of shear stresses across the interfaces. Detailed mathematical features of the element, together with simplified constitutive relations to define its behavior, can be found in recently published work of the authors [Tzamtzis 1992, Tzamtzis 2002].

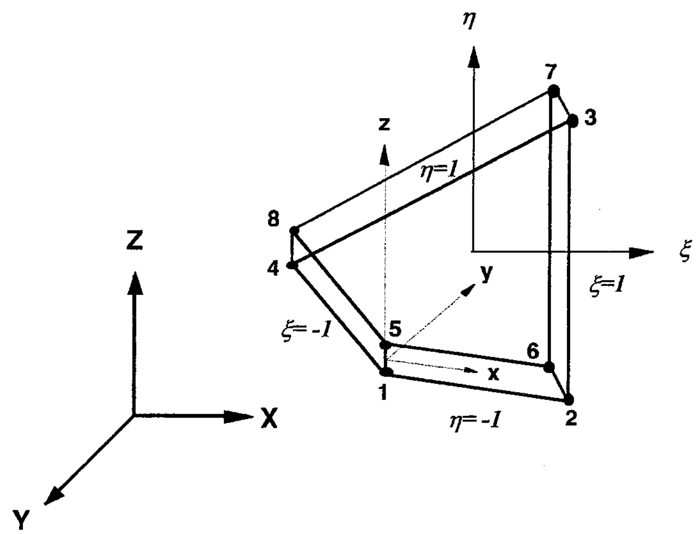

Fig. 1. Geometry of the 3-D interface element

The constitutive model of the interface element reflects the limiting value of the shear stress/normal stress ratio at the interface. Insertion of the element leads to the satisfaction of the compatibility condition at the interface and to the representation of the strain energy contributed by the discontinuous interface [Page 1978]. The stiffness properties of such an interface element can be derived from tests that can simulate the transfer of shear stresses. For wall thickness t_w and mortar joint thickness t_m, the normal and shear stiffness required to define the material property matrix of the element, can be represented by the following expressions [Page 1978]:

$$k_{sx} = k_{sy} = \frac{G\,t_w}{t_m}; \quad k_{nz} = \frac{E\,t_w}{t_m} \qquad [1]$$

in which E and G are the instantaneous tangent elastic and shear moduli at the particular value of normal and shear stress being considered. The non-linear behavior of the joints can therefore be treated by assigning the joint properties corresponding to the level of stress obtained from the last load step in a step-by-step loading analysis procedure.

The failure criteria of a joint depend mainly on the relative magnitudes of the normal and shear stresses present in the joint. The relationship between the normal stress in a joint and its ultimate shear strength can be obtained from tests on masonry prisms with the load inclined to the bed joints. A failure criterion of this type, obtained by Page [Page 1978], has been adopted in the present analysis. When used in the analytical model, this criterion allows progressive joint failure to occur. If the criterion for the failure of a joint element is violated, the element properties are modified and the problem is resolved. If failure occurs due to a normal tensile stress in the joint, the joint element stiffness is set equal to zero in both the normal and shear directions. If failure occurs under a combination of compression and shear stresses, the stiffness of the joint in the normal direction is assumed to remain unchanged, and a reduced value for the shear stiffness is allocated, depending mainly on the compression stress in that joint. As the normal stress diminishes, the residual shear stiffness is assumed to reduce to zero under a condition of pure shear. Introducing the above constitutive laws in the finite element program, the inelastic behavior of the mortar joints can be represented.

The proposed interface element is three-dimensional and it does not appear to have been used in the non-linear analysis of three-dimensional masonry structures subjected to static or dynamic loading. In order to test the element's performance, it would be logical to use it to analyse a truly three-dimensional problem for which analytical or experimental results are known or available. Unfortunately, no such work, analytical or experimental, appears to have been done to date with which present results of a three-dimensional finite element analysis could be compared. Reliable results of tests on two-dimensional masonry walls are, on the other hand, available, under static and dynamic loading conditions. For this reason, two-dimensional masonry walls analysed by other investigators have been chosen as the test problem, in both static and dynamic analyses. A three-dimensional formulation has been used instead, to demonstrate the level of accuracy the proposed interface element is capable of.

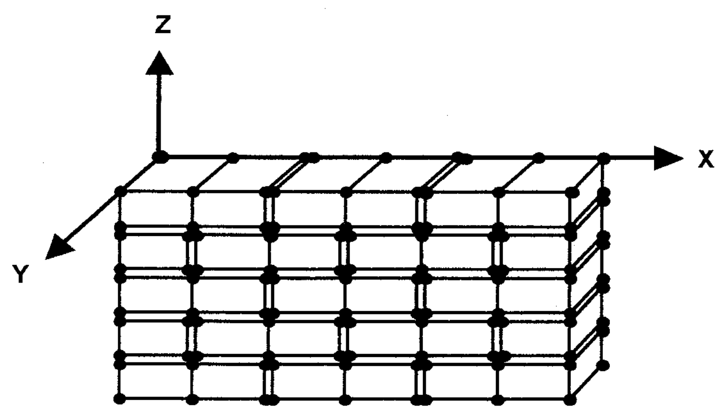

Fig. 2. Typical finite element subdivision

A typical discertization of a masonry wall is shown in Figure (2). It was discertized using 8-noded isoparametric 'brick' elements to represent the bricks, while the mortar joints (shown exaggerated), where represented by three-dimensional interface elements (Figure 1).

Static Analysis

The problem and method of analysis

The analysis of an unreinforced masonry wall panel subjected to an in-plane static bending loading, for which experimental and analytical results are available in the literature, is presented here. Bending tests on a masonry deep beam have been performed by Page, to verify his two-dimensional finite element model for the analysis of masonry walls subjected to in-plane loading. Since the present study deals with the extension and development of a similar model in three-dimensions, the experimental results obtained by Page [Page 1978] were used here to verify the results obtained from the proposed three-dimensional model. The test arrangement of the masonry wall beam used for the experiments is given in [Page 1978].

Incremental analysis is able to give full illustration of the loading processes and has obvious advantages in describing complicated processes such as loading-unloading-reloading and so on, and for this reason it has been employed in the present study. The element types described have been incorporated into an incremental finite element computer program especially developed for the analysis. The load was applied incrementally in terms of equivalent displacements at the relevant nodes, in order to avoid numerical instability. At a given load level, iteration was continued until stresses generated by the loads satisfied the yield or failure criteria within prescribed tolerances. Convergence was taken to have been achieved when the displacement increment vector, from one iteration to the next, was less than the chosen tolerance. A further increment of load was applied once convergence was achieved, and the process repeated as required. Non-linear material characteristics, for both bricks and joints, were modeled using the *initial stress* method. Inelastic behavior of the bricks was also introduced in the finite element analysis.

Non-linear finite element analysis of three-dimensional masonry prisms is a complex and time-consuming exercise, which relies on a detailed knowledge of the stress-strain relationships of both the masonry units under a multi-axial state of stress. Since such information is difficult and often not possible to provide, elastic-perfectly plastic behavior has been assumed in the present analysis for simplicity. The failure criterion used for the brick, is the Mohr-Coulomb criterion, which is best suited for brittle materials with properties similar to concrete. This criterion is also used because of its inherent simplicity and convenience.

Results - Verification of the model

Vertical stress distributions at level A-A, obtained from experiments performed by Page [Page 1978], are compared in Figure (3) to the results obtained using the three-dimensional finite element model developed. Stress distributions at two levels of applied load, P, are shown.

Results obtained from a conventional finite element analysis, with the masonry modeled as a continuum with average properties and isotropic elastic behavior, are also included. The computed progression of cracks, as predicted by the finite element model, is also shown in Figure (3), with P increasing from 40 KN to 60 KN. Clearly, there is a good agreement between the results of the present inelastic analysis and the experimental results of Page [Page 1978], particularly in view of the inherent variability of actual material properties, joint strengths and the various simplifications that have been made in the proposed analytical model (stress-strain relationship, failure criterion, etc.). It is also clear that the elastic solution deviates grossly from both experimental and the inelastic results, and this confirms that material behavior is significantly non-linear, especially that of the mortar joints.

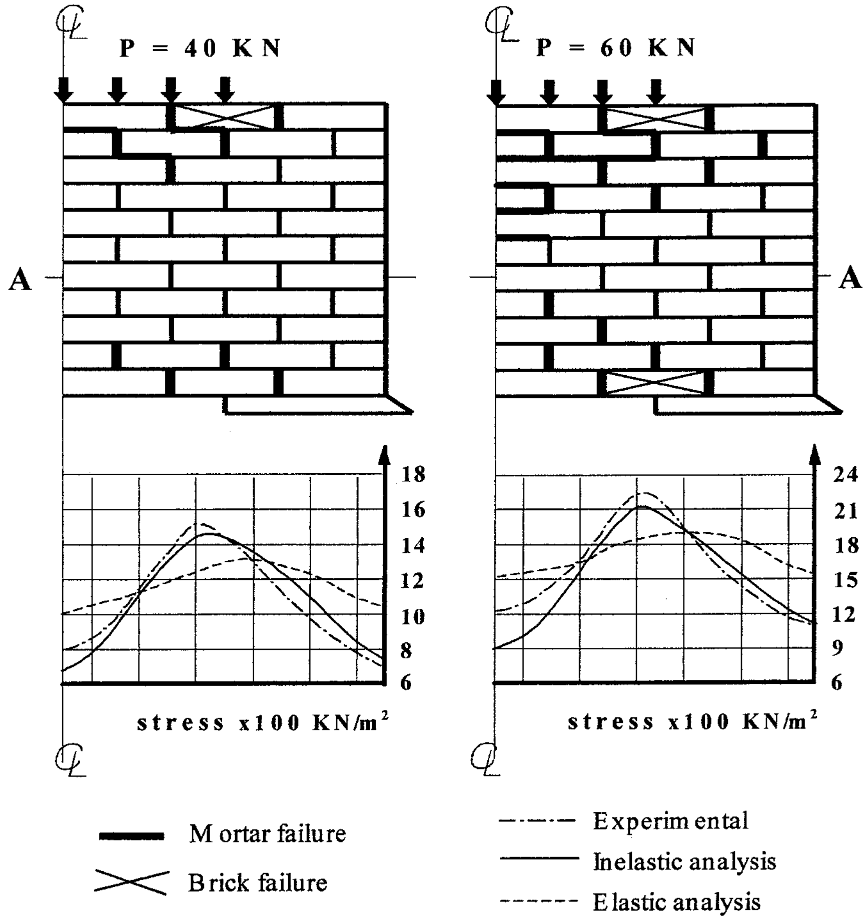

Fig. 3. Comparison of present analytical results with the experimental results of Page [Page 1978]

Seismic Analysis

The problem and method of analysis

The calculation of the dynamic response of a discontinuous system subjected to time-varying loads such as blast, impact or earthquake loading is generally a complicated process, especially if the non-linear behavior of the system is simulated. Given that the majority of earthquake fatalities world-wide by far are caused by the failure of masonry structures, and that masonry is probably the least understood of all construction materials, the need for developing an accurate model of the seismic behavior of masonry structures cannot be over-stated. Unfortunately, information on the material properties of masonry under dynamic/seismic loading is both complex and scarce. At present there appears to be very little knowledge about the dynamic properties of masonry, especially in the non-linear regime. The problem is further compounded by the unavailability of results, experimental or analytical, with which present results could be compared. Indeed, to the best of the authors knowledge, the only relevant work available with which present results could be compared, is the shaking table test on a brick wall, shown schematically in Figure (4), due to Mengi and McNiven [Mengi 1986, Mengi 1989, McNiven 1989].

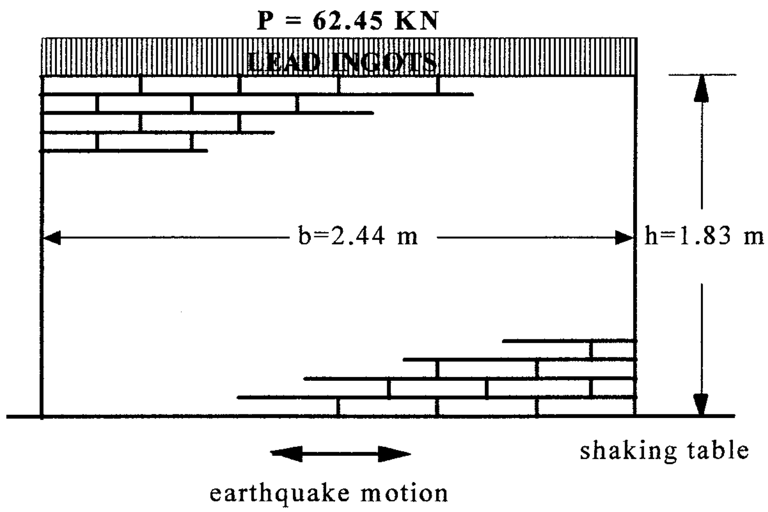

Fig. 4. Schematic diagram of the seismic test setup

The finite element model described, has been used to predict the in-plane response of a masonry wall to a given earthquake excitation. The bricks in Figure (4) were modeled with 8-noded isoparametric 'brick' elements, and the joints between adjacent bricks with the interface element of Figure (1). The base excitation input to the system was taken to be the modified El-Centro earthquake record shown in Figure (5), used by Mengi and McNiven [Mengi 1986] for experiment 11 in their study. The modification involved the reduction of the time-scale by a factor of $\sqrt{3}$ so that it was possible to study the wall behavior for a wider range of frequencies.

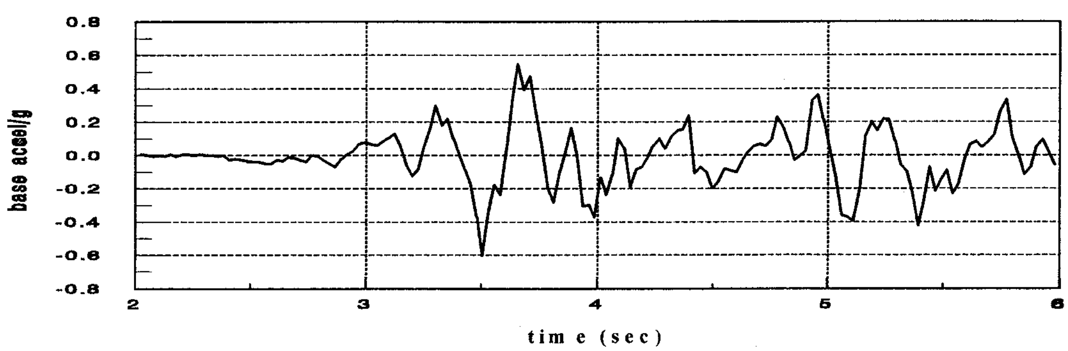

Fig. (5). Seismic record of the modified El Centro earthquake (taken from [Mengi 1986])

Since material behavior under dynamic loading is very complex and experimental information is scarce, attemps to perform an analysis of a dynamically-loaded structure must include an idealised material model where some compromises can be made. The simplest stress-strain law has been implemented in the present finite element analysis and involves elastic-perfectly plastic material behavior. Isotropic conditions have been assumed with the following material properties: assuming that the condition G/E=0.5

obtained by Mengi and McNiven [Mengi 1986] for the dynamic case holds in general, the value of the Poisson ratio has been taken as zero, and the value of the Young's modulus (considered to be the same for both brick and mortar) has been taken as $E=3429$ kp/cm^2.

Both the Wilson-theta and Newmark average acceleration methods (i.e. $\alpha=1/4$ and $\delta=1/2$) have been employed in the present study for computing system response. The optimised size of the time-step, Δt, was taken as 0.025 sec, and the value of theta used in the Wilson-theta method as 1.4 in order to ensure the unconditional stability of the solution. The size of errors, which may be introduced by any numerical integration scheme, depends on the characteristics of the dynamic loading and the time step. The general nature of the computational errors may be expressed in terms of an artificial change of period and a reduction of amplitude. Period-elongation and amplitude-decay, caused by the numerical integration errors, can be important and should be taken into account when comparing the results of the analysis.

Very limited information is available on damping in linear solid mechanics problems, and even less data is available on damping in non-linear situations, especially for materials subjected to seismic loads. However, since the effect of damping is small compared to those of inertia and stiffness, the damping matrix [C] may be represented by simplified expressions. In this analysis it has been assumed, therefore, that the damping matrix is proportional to the mass and stiffness matrix. This is known as Rayleigh damping and we have

$$[C]=a[M]+b[K] \qquad [2]$$

in which a and b are proportionality constants selected to control the damping ratios of the lowest and highest modes expected to contribute significantly to the response. For the problem under consideration, the value of a was taken as zero, and the value of b was taken as 0.24 [Mengi 1986]. Rayleigh modelling of damping is rather poor since constants a and b are fixed for all modes of vibration. The effect of this simplification on the response history of the structure has to be acknowledged.

The stress level in the structure before the application of the seismic base excitation was assumed to be due to extra loading on top of the wall (Figure 4). The influence of this mass on the seismic behavior of the wall was considered by taking into account the mass of the lead ingots attached to the top face of the wall. Prior to the seismic analysis, the initial stresses were calculated using a static finite element program. The stress state for every Gauss integration point was recorded and added to the input data for seismic analysis.

Response results

The experimental response of the brick masonry wall of Figure (4), subjected to the El Centro earthquake motion (Figure 5), was obtained from an experimental program on a shaking table performed by Mengi and McNiven [Mengi 1986]. It has been used here as the basis for comparison of the results. This is given in Figure (6), and is compared with the system response as predicted by the present finite element method using both Wilson-theta and Newmark's implicit algorithms. Despite the various simplifications and assumptions that have been made in the present finite element analysis (material properties, stress-strain behavior, failure criterion, Rayleigh damping, etc.), the computed response using both the algorithms are in very good agreement with the experimental response. Unfortunately, not all the material properties necessary to define the present finite element model could be found; therefore, some of the properties had to be assumed. This may be the reason for the discrepancy between test and present results. In addition, experimental conditions could not be reproduced accurately, since the test specimen was composed of two parallel walls connected to a steel base frame which in turn was bolted to the shaking table. The finite element model used in the present analysis simulates only one of the walls with half of the imposed weight placed on it.

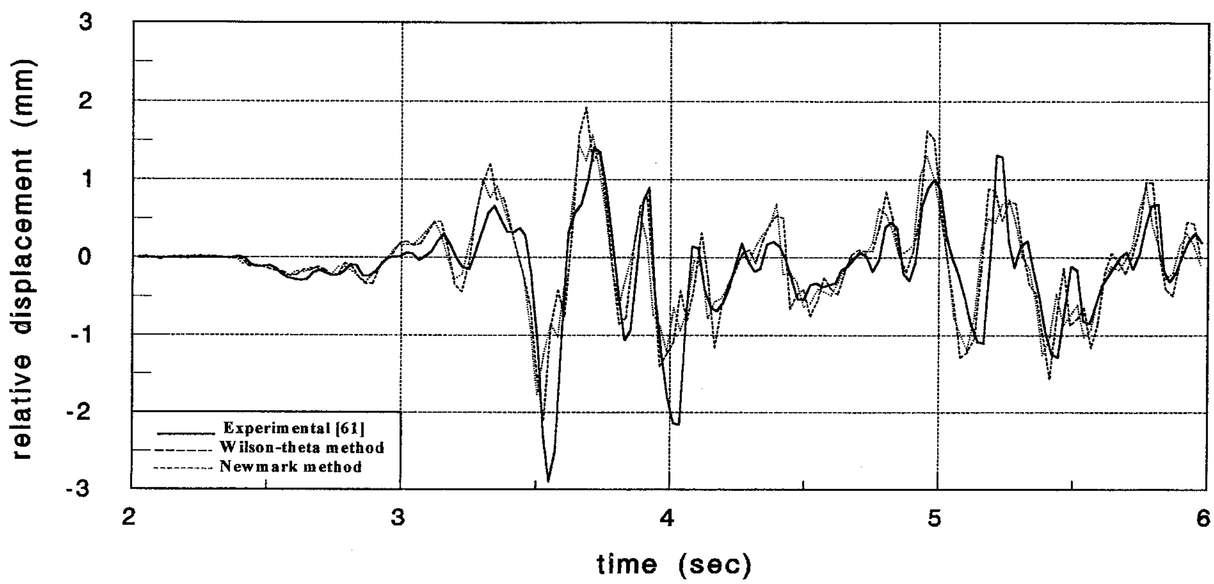

Fig. 6. Comparison of the computed seismic responses of the wall of Fig. (4) to the excitation of Fig. (5)

Summary and Conclusions

A non-linear finite element model for the analysis of masonry walls, subjected to static and earthquake loading conditions, has been presented in this study. The model developed is three-dimensional and considers masonry as a two-phase material, treating bricks and mortar joints separately, thus allowing for non-linear deformation characteristics and progressive local failure of both bricks and mortar joints. The influence of the mortar joints has been taken into account by using interface elements to simulate the time-dependent sliding and separation along the interfaces.

Stress distributions from an in-plane bending test on a masonry panel performed by other investigators, were reproduced to a reasonable degree of accuracy by the present finite element model. The accuracy and potential of the proposed three-dimensional model were also demonstrated by using the model to predict the non-linear response of an unreinforced masonry wall to in-plane earthquake excitations. Analytical and experimental solutions available in the literature have been employed to verify the results obtained from the present finite element model, showing that it is capable of accurate replication of the masonry's behavior.

The use of interface element in the finite element analysis of masonry structures, employed to simulate the actual joint behavioral features of the system, may eventually lead to a more rational design of these structures. The model adopted offers a more realistic alternative to an analysis that assumes masonry as continuum with average properties, and it has a potential as a research tool since the properties of the masonry components can be varied individually and their significance to the overall behavior of the masonry assemblage investigated.

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