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FINITE ELEMENT ANALYSIS OF MASONRY STRUCTURES: PART I – REVIEW OF PREVIOUS WORK

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Abstract

In this study a three-dimensional microscopic finite element model is proposed to predict the nonlinear behavior of masonry structures when subjected to static loading and/or dynamic excitations. Owing to its length, the study is divided into two parts. This part contains a comprehensive review of previous work, summarizing several methods and finite element models developed for the static and dynamic analysis of masonry structures. The development of the proposed microscopic model is then presented in detail in Part II of the study, where the accuracy and potential of the model is also demonstrated. Analytical and experimental solutions available in the literature are employed to verify the results obtained from the application of the proposed finite element model, showing that it is capable of a high degree of accuracy.

Keywords: masonry structures, finite element analysis, literature review, interface element

Introduction

Analytical and experimental studies on the behavior of masonry walls when subjected to in-plane static or dynamic loads have been the focus of activity of a number of investigators for many years. Early studies concentrated mainly on experimental investigations by necessity, and these have been hampered by the large number of variables that must be considered in the overall behavior of the masonry. The great number of the influencing factors, such as dimension and anisotropy of the bricks, joint width and arrangement of bed and head joints, material properties of both brick and mortar, and quality of workmanship, make the simulation of plain brick masonry extremely difficult.

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A review of the available literature on the finite element modeling of masonry reveals that two-dimensional plane stress formulations have mainly so far been adopted for the analysis of masonry structures. Further, in the absence of a suitable model to represent its behavior, masonry was assumed to be an isotropic elastic continuum in the past. Consequently, the influence of the mortar joints acting as planes of weakness, could not be addressed. Indeed, it is only recently that analytical procedures, which account for the non-linear behavior of masonry under static loads, have been developed. An extension of these procedures to static and dynamic/seismic response analysis of three-dimensional masonry structures is presented in this study. In particular, a 3-D nonlinear microscopic finite element model for static and dynamic analysis of unreinforced masonry structures is presented, together with comparison of its performance against available experimental results.

Owing to its length, the study is divided into two parts. This part contains a comprehensive review of previous work, summarizing several methods and finite element models developed over the years for the analysis of masonry structures, both in the static and dynamic regime. The review is however limited to the finite element modeling of unreinforced masonry, since this is the subject of the present study. The development of the proposed 3-D nonlinear microscopic model is then presented in detail in Part II of the study [Tzamtzis 2003]. The potential and accuracy of the model is demonstrated by using it to predict the nonlinear response of unreinforced masonry wall assemblages when subjected to static loading and dynamic excitations. Analytical and experimental solutions available in the literature are employed to verify the results obtained from the application of the proposed finite element model, showing that it is capable of a high degree of accuracy.

Models for masonry

Several investigators have dealt with the structural behavior of masonry in recent years using the finite element method. Most analyses have considered masonry to be an assemblage of bricks and mortar with average properties. Isotropic elastic behavior has been assumed to simplify the problem [Rosenhaupt 1965, Saw 1974], ignoring the influence of mortar joints acting as planes of weakness. Assumptions like these were useful in predicting deformations at low stress levels, but not at higher stress levels where extensive stress redistribution caused by non-linear material behavior and local failure would occur. Material models, based on average properties and with the influence of mortar joints ignored but including the possibility of local failure, have also been developed by some investigators [Ganju 1977, Samarasinghe 1982].

Dhanasekar et al. [Dhanasekar 1984] proposed a non-linear finite element model for solid masonry based on average properties derived from biaxial tests on brick masonry panels. The model is capable of reproducing the effects of material non-linearity and progressive local failure, but the masonry is modeled as a continuum with average properties, with each element in the finite element subdivision including several bricks and joints. The model, therefore, has limitations when local effects are important and cannot be used, for example, to predict the behavior of masonry subjected to concentrated loads where local stresses and stress gradients are high. Other investigators have produced more refined models, including elastic analyses with bricks and mortar joints being modeled separately

[Smith 1970, Smith 1972, Ali 1985]. A method that accounts for the non-linear behavior of masonry, considering masonry as a two-phase material, was first developed and applied to solid masonry by Page [Page 1978] and to grouted and hollow concrete masonry by Hegemier et al. [Hegemier 1978]. Ali & Page also used the method to study the non-linear behavior of masonry subjected to concentrated loads [Ali 1987, Ali 1988]. A theoretical parametric study of the behavior of storey height solid masonry walls, subjected to concentrated loads, has been performed by Ali and Page [Ali 1988] using multi-level substructuring and mesh-refinement techniques for the efficient modelling of the walls. In the parametric study of the wall behavior, the influence of the loaded area ratio, the load location, and the wall geometry have been considered and design relationships have been proposed.

The majority of the proposed constitutive models for masonry, therefore, can be classified in two categories: (i) the *one-phase* material models, treating masonry as an ideal homogeneous material with constitutive equations that differ from those of the components; and (ii) the *two-phase* material models where the components are considered separately to account for the interaction between them. The constitutive models of the first category are relatively simple to use and require less input data, and the failure criterion has normally a simple form. On the other hand, their constitutive equations are relatively complicated and are suitable at best for the study of the global behavior of masonry. The 'two-phase' material models are relatively costly to use due to the great number of the degrees of freedom, require more input data, and their failure criterion has a complicated form due to the brick-mortar interaction. The constitutive equations of the components have normally a simple form, on the other hand, and they are suitable for the study of local behavior of masonry.

Lourenco and Rots have recently presented in detail the two approaches (Micro- and Macro-modeling) for the analysis of masonry structures [Lourenco 1993, 1995]. Conclusions regarding the relative merits of various models and algorithms should be based on direct comparison of the methods, when applied to identical problems. Depending on the kind of problem therefore, the degree of accuracy required versus the simplicity desired, one could use the appropriate model and method to analyse the problem in hand.

Material behavior

Static loading

The development of improved models of material behavior was made possible by the increased sophistication of numerical methods of stress analysis. A complete model requires the elastic properties of masonry, a yield criterion, inelastic stress-strain relations, and a failure criterion. Most of the studies to date have concentrated on finding a failure criterion rather than on studying the deformation characteristics of the material, and they have been restricted to tests under monotonic loading conditions. Masonry elements are subjected to biaxial states of stress produced by in-plane loading, and only recently there have been attempts to study the material properties of brick masonry subject to biaxial stress states.

Information on the stress-strain relations for brick masonry is limited and restricted to data on the elastic properties. The average elastic properties of brick masonry have been reported by Dhanasekar et al. [Dhanasekar 1982] after a series of tests on masonry panels. Extensive tests were conducted on grouted concrete masonry by Hegemier et al. [Hegemier 1978], who determined the elastic properties and the failure surface under biaxial stress state.

The failure of masonry under uniaxial compression, combined shear and compression, and tension, has been studied extensively in the past by many investigators [Khoo 1973, Grimm 1975, Hendry 1978, Smith 1970]. These failures all represent particular points on the general failure surface. The development of a general failure criterion for masonry is difficult, because of the difficulties in developing a representative biaxial test and the large number of tests involved. Yokel and Fattal [Yokel 1976] discussed the problem with reference to the failure of shear walls, and, in their study, various failure hypotheses were compared with the results of tests on single-wythe clay brick walls.

Page [Page 1980] proposed a failure surface for brick masonry subjected to biaxial tension-tension, using a non-linear finite element model accounting for joint failure. He also reported experimentally derived failure surfaces for half-scale brick masonry subjected to compression-compression and tension-compression stress states [Page 1981, Page 1982, Page 1983]. A complete failure surface was developed later by Dhanasekar et al. [Dhanasekar 1985] as an extension of these experimental results, and an approximate method of establishing a conservative failure surface from a reduced number of uniaxial and biaxial tests was proposed.

Samarasinghe and Hendry [Samarasinghe 1980] also obtained a failure surface from tests on one-sixth scale of brick masonry subjected to biaxial tension-compression. The shape of both these failure surfaces was found to be critically dependant on the bed-joint orientation and the relationship between the shear and tensile bond strengths of the mortar joints. The influence of the orientation of the applied stresses to the joints has also been investigated for grouted and ungrouted concrete masonry by Hamid et al. [Hamid 1981] and failure criteria were proposed as a generalised form for masonry taking into consideration its anisotropic nature as a composite material.

The tensile strength of concrete masonry has also been investigated by the same authors [Drysdale 1979], and tension failure criteria for unreinforced concrete block masonry have been developed [Drysdale 1984] which account for strength variation due to the anisotropic nature of masonry as a composite material. Ganz and Thurlimann [Ganz 1983] suggested a failure surface for brick masonry constructed from highly perforated bricks in terms of the direct stresses parallel and perpendicular to the bed joint, and shear stress on the bed joint. The failure surface was defined by four separate functions of these stresses, related to four distinct failure modes.

Non-linear stress-strain relations for brick masonry have been derived by Dhanasekar et al. [Dhanasekar 1985] from the results of a large number of biaxial tests on square panels with various angles of the bed joint to the principal stress axes. It has been found that although the initial elastic behavior is close, on average, to isotropic, the non-linear behavior is strongly influenced by joint deformations and is best expressed in terms of stresses and strains referred to axes normal and parallel to the bed joint orientation.

Comparatively little research has been reported on the behavior of brick masonry under cyclic compressive loading. The effects of repeated compressive loading are particularly applicable to brick masonry structures having large live load to dead load ratio. Recently, an investigation of the behavior of brick masonry under cyclic, uniaxial and biaxial compressive loading has been presented by Naraine and Sinha [Naraine 1988, Naraine 1989, Naraine 1989]. They established that the uniaxial and biaxial stress-strain history possess a locus of common points where the reloading part of any cycle crosses the unloading part of the previous cycle which can be used to define the permissible stress levels for brick masonry under cyclic loading. A generalised approach is also proposed by the same authors [Naraine 1992] to determine the envelope, common-point, and stability-point curves on the absolute stress-strain coordinate system, using a general stress-strain equation for each principal stress ratio.

Asteris and Tzamtzis [Asteris 2002] have recently developed a methodology for the non-linear 'macroscopic' analysis of unreinforced masonry walls under biaxial stress state using the finite element method. One of the advantages of the proposed material model is that average properties, which include the influence of both brick and joint, have been used. This means that a relatively coarse finite element mesh can be used (with any element typically encompassing several bricks and joints), giving considerable computational advantages when analyzing large wall panels. The methodology focuses on the definition / specification of a general anisotropic (orthotropic) failure surface of masonry under biaxial stress, using a cubic tensor polynomial, as well as on the numerical solution of this non-linear problem. The characteristics of the polynomial used, ensure the closed shape of the failure surface which is expressed in a unique mathematical form for all possible combinations of plane stress, making it easier to include it into existing software for the analysis of masonry structures [Asteris 2000, Syrmakezis 2001].

Lourenco and Rots have also recently proposed an anisotropic failure criterion for masonry, suitable for numerical implementation [Lourenco 1998, 2000]. The authors conducted a parameter estimation and validation study, and the continuum model for masonry developed, has been implemented into DIANA computer program.

Dynamic loading

The behavior of masonry walls under earthquake forces is of primary importance for construction in earthquake-prone areas. Devastating damage to masonry structures in the last thirty years due to earthquakes has caused engineers to consider masonry as a structural material, to recognise its weaknesses and to consider ways to overcome these so that such damage can be significantly reduced in the future. Experimental and analytical research on the seismic behavior of masonry structures during the last fifteen years has revealed that their earthquake performance can be significantly improved by properly reinforcing against tensile stresses that cause brittle failure because of the low tensile strength of masonry.

The state of knowledge concerning shear strength and shear load-displacement behavior of masonry is far less advanced than that concerning behavior in compression, even though shear is the dominant mode of failure observed in many masonry buildings subjected to lateral loading due to earthquakes, wind or other causes [Kariotis 1985]. Most of the research conducted to date with regard to masonry shear behavior has been limited to determining the peak shear strength and the factors that affect it. Additional information on the shear stiffness for both initial and repeated loading states, peak and residual strength values, repeated shear reversals, and dynamic effects are required in order to construct analytical models to simulate response under realistic seismic loading conditions. At present there appears to be very little knowledge about the dynamic mechanical properties of masonry.

The horizontal bed joint shear failure mode and the shear load-displacement behavior of unreinforced brick masonry during static and cyclic loading have been examined by Atkinson et al. [Atkinson 1989]. Results of tests by various researchers [Meli 1973, Priestley 1974] on reinforced masonry cantilever walls, subjected to in-plane lateral loads, indicate the existence of two basic failure modes: shear failure, which is characterised by diagonal cracking of the masonry along lines of principal tensile stresses in the wall plane; and flexural failure, which is characterised by either yielding of tension steel, followed by crushing of the masonry, or by crushing of the masonry alone at the compression toe.

The behavior of masonry piers under in-plane cyclic loading has been reported by many investigators in relation to the seismic design of buildings. These experimental studies are aimed at modifying and improving the existing codes for masonry structures. Mayes et al. [Mayes 1976, Mayes 1976] were concerned with the strength, failure modes, and cyclic shear behavior of masonry piers, and Hidalgo et al. [Hidalgo 1978, Hidalgo 1979] have studied the cyclic behavior of masonry piers subjected to load reversals. They tested various fixed-end masonry piers, some of which developed high bearing stresses leading to a shear mode of failure, and observed that the structural behavior of the specimens failing in shear was much more complicated than those failing in flexure.

Gulkan et al. [Gulkan 1979, Gulkan 1979] tested single-story masonry houses on a shaking table and investigated their behavior by subjecting them to earthquake excitations. Manos et al. [Manos 1983] studied the effect of combined in-plane and out-of-plane action on the earthquake performance of the same models used by Gulcan et al., as well as to vertical input motion. Priestley [Priestley 1985] made an attempt to explain the behavior of unreinforced masonry walls under seismic loading, with particular emphasis being given to face-load response. The response of unreinforced masonry walls to out-of-plane (face-load) seismic excitation is one of the most complex and little understood areas in seismic analysis.

Extensive dynamic analyses and shake-table testing of face-loaded walls, performed by Kariotis et al. [Kariotis 1985, Kariotis 1985], indicated that the walls could sustain levels of excitation acceleration far greater than that predicted by elastic or ultimate strength calculations. They found that a correlation could be found between the strength of face-loaded walls and the spectral velocity of the input acceleration. This indicates that energy considerations are important. An analytical model was developed by Button et al. [Button 1992] to predict the out-of-plane seismic behavior of reinforced masonry walls. The model was validated by comparing its predicted response with those obtained from full-scale dynamic tests on clay-brick walls performed by Blondet et al. [Blondet 1990], and tests on concrete-block walls performed by Adham et al. [Adham 1990].

The first attempt in developing a mathematical model to predict the response of masonry walls to dynamic excitation was made by Sucuoglu et al. [Sucuoglu 1982, Mengi 1984, Sucuoglu 1984]. Two mathematical models were presented by the authors for predicting the linear in-plane dynamic behavior of masonry walls. The first, called 'mixture model', recognises the two-material composition of masonry and predicts its response accurately for a wide range of frequencies. The theoretical framework of the model was established by Mengi and McNiven [Mengi 1979]. The second, called the 'effective modulus' model, is simpler and accurate for a smaller frequency range. The study was presented in three stages: experimental observations, selection of the mathematical model, and the determination of model parameters through optimisation analysis.

Later, a model for predicting the non-linear in-plane behavior of clay brick masonry walls, when subjected to dynamic excitations, was proposed by the same authors [Mengi 1986] by extending the previously developed linear model to cover behavior in the non-linear range. In the study of the mechanical properties of masonry, it has been found that the values established when masonry is responding to dynamic forces are radically different from those when the masonry is responding to static forces. These values remain constant until cracking begins, after which the shear modulus decreases and the damping coefficient increases. Results and optimisation show that these changes are almost linear. The same study was published again by the authors in two parts [Mengi 1989, McNiven 1989]: Part 1 was devoted to a description of the appropriate experiments and to the development of the mathematical model, including use of experimental data. In Part 2, the model was completed by establishing the 'parameter functions' appearing in it.

Final Considerations

A comprehensive review of previous work on the finite element modeling of unreinforced masonry structures, both in the static and dynamic regime, has been conducted in this part of the study. It is generally agreed that refined models such as nonlinear microscopic and anisotropic models are adequate to analyze simple structures like masonry shear walls subjected to in-plane loading, whereas more simplified models like nonlinear macroscopic or linear elastic 3-D models are suitable to analyze entire masonry buildings. It is also revealed that two-dimensional analyses with plane stress formulation have mainly so far been adopted for the finite element analysis of masonry structures. Further, in the absence of a suitable model to represent its behavior, in the past masonry was assumed to be an isotropic elastic continuum; consequently, the influence of the mortar joints acting as planes of weakness, could not be addressed. Indeed, it is only recently that analytical procedures, which account for the non-linear behavior of masonry under static loads, have been developed. An extension of these procedures to static and dynamic/seismic response analysis of masonry walls in three-dimensions is presented in Part II of this study [Tzamtzis 2003], where the development of a 3-D nonlinear microscopic finite element model, suitable for static and dynamic analysis of unreinforced masonry structures, is presented.

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